1. **Intensity Dependent Refractive Index: Beam Deflection**

As briefly discussed in the class, among the sensitive methods of nonlinear refraction measurements is the beam deflection technique. As shown in the figure, a pump beam (with a Gaussian spatial profile) induces a refractive index profile in the nonlinear material. A weak probe beam—with a diameter much smaller than that of the pump—centered at a distance $x_0$ away from the pump beam will be therefore deflected by this index gradient.

Assume a pump irradiance $I(r)=I_0 \exp\left(-2r^2/w_0^2\right)$ and optical Kerr effect: $n=n_0+n_2 I$. We also know that a ray traveling through a thin material of transverse gradient index $n(r)$ is deflected by an angle $\phi=\nabla n(r)L/n_0$ where $L$ is the thickness of the sample and $n_0$ is the linear refractive index.

a) What is the optimum position $x_0$ for which the maximum deflection occurs?

b) What is the maximum deflection angle?

The deflection of the beam is measured using a “bi-cell” detector in the far field as shown in the figure. In that case, the measured normalized signal is $S=(V_1-V_2)/(V_1+V_2)$.

c) Show that $S=\sqrt{8\pi} \frac{w_0}{\lambda} \phi$

where $w_0$ the probe beam radius at the sample and $\lambda$ is the wavelength.

d) Write $S$ in terms of the maximum on axis phase shift $\Delta \Phi_0=(2\pi/\lambda)n_2 I_0 L$ (i.e. $S=K \Delta \Phi_0$). Derive an expression for $K$ and compare the sensitivity of this technique with that of z-scan where the normalized peak-to-valley transmittance signal is given by $\Delta T_{pv}=0.4 \Delta \Phi_0$.

![Beam deflection diagram](image)

2. **Self-Phase Modulation (SPM)**

a. A laser pulse with an intensity profile $I=I_0 \text{Sech}^2(t/\tau_0)$ having $\tau_0=200$ fs and $\lambda=500$ nm, and $I_0=1$ GW/cm$^2$ is coupled into a silica fiber having an instantaneous (ultrafast) $n_2=2\times10^{-16}$ cm$^2$/W. **Estimate** the required length of fiber ($L$) for the spectrum of the pulse to broaden (due to SPM only) to $\approx 5$ times its original value.

b. Using linear dispersive elements such as grating pairs, we can compress the exit pulse in part (a) to a transform-limited pulse having a width $\approx 1/\Delta \omega$. **Estimate** the resultant compressed pulse width.

c. If the pulse in (a) propagates in a nonlinear medium with $n_2=1\times10^{-14}$ cm$^2$/W but with a relaxation time $\tau=2$ ps, what is the required length to double its spectrum. **Qualitatively**, describe the transmitted spectrum as compared to that of part (a).