You are asked to perform a photo-luminescence experiment where a pump laser at wavelength $\lambda_p$ is incident through a dichroic beam splitter and is focused onto the sample with a lens with known f# ($=f/D$). The luminescence (centered at $\lambda_f>\lambda_p$) is then collected with the same lens and is imaged into a detector or monochrometer for analysis. All lenses are AR coated.

a) Knowing the internal PL power $P_f$, calculate the collected (external) PL power $(P_{ex})$ at the detector. Write $P_{ex} = \eta_e P_f$ and show that $\eta_e$ (the extraction efficiency) can be approximated as

$$\eta_e \approx \frac{1}{4n(n+1)^2} \times \frac{1}{(f\#)^2}.$$  

where $n$ is refractive index of the sample. *Hint: This is the fraction of solid angle (inside the sample) subtended by the lens after exiting. Ignore the reflection of the fluorescence from the bottom surface.*

b) The detector system can be characterized by the following parameters: load resistance ($R_L$), capacitance (C), quantum efficiency $\eta_q$ and gain $G$. The dominant noise is the Johnson noise of the load resistor at temperature $T$. Write the SNR of the system in terms of $P_f$, $\eta_q$, $\eta_e$, $\eta_i$, $R$, $C$, and other known parameters and constants.

c) Consider now that the sample under study is bulk GaAs at $T=300$ K ($E_g=1.42$ eV, $n=3.6$). Assume an internal luminescence power of 10 $\mu$W. Assume the mean luminescence frequency is $\nu_f = \nu_p = E_g/\hbar$. The f# of the lens is 4.

i) What is the SNR if we use a regular photodiode (PD) $\eta_q=1$, $G=1$ terminating into an oscilloscope with $R_L=1$ M$\Omega$, $C=5$ pF at $T=300$ K?

ii) What is the SNR in (ii) if we replace the PD with an APD with $G=100$ and excess noise factor of 10? What gain is necessary for making the detection shot-noise limited?