Laser Physics-I (PHYC/ECE 464), *Fall 2022*

Homework #7, Due Wed. Oct. 19

1. Consider a pressure-broadened gaseous two-level medium with the following property:

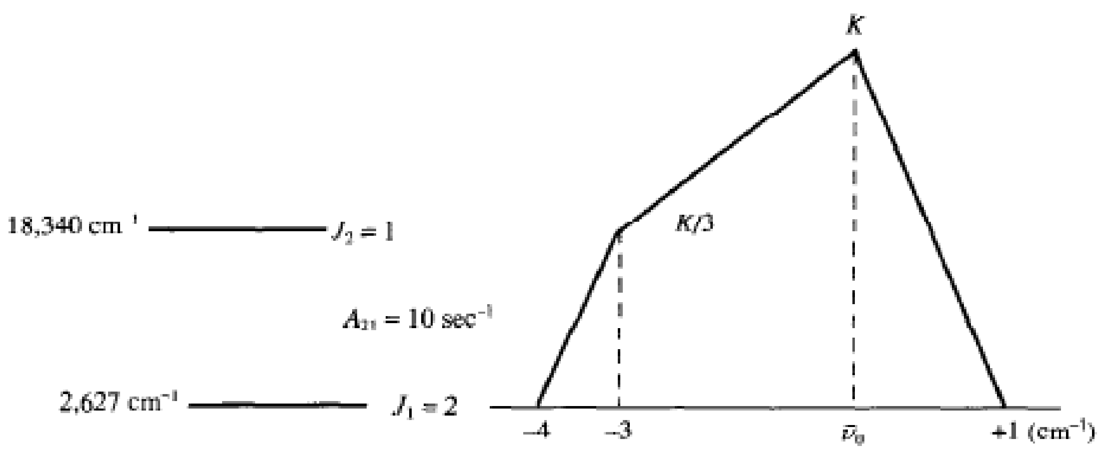
- *Spontaneous emission lifetime:* $\tau_{sp}=1\ \mu s$
- *Homogeneous linewidth* $\Delta\nu_h=1.5$ *THz*
- *Line center wavelength:* $\lambda_0= 5\ \mu m$
- *Molecular density (concentration):* $N_{total}= 2.5\times10^{19}\ cm^{-3}$
- *Non-degeneracy factors:* $g_1=5$, $g_2=1$

(a) What is the absorption coefficient $\alpha(cm^{-1})$ at the line center (5 μm) when all the molecules are in their ground state (level 1)?

(b) What fraction of the molecules needs to be excited into level 2 in order to make this gas transparent (i.e. the onset of gain) at 5 μm?

2.

7.3. The spontaneous emission profile from a certain transition can be approximated by the shape shown below.



(a) What is the stimulated emission cross section?
(b) What is the absorption cross section?

3

7.5. Consider a transition of 5000 Å with a width of 1 Å, a cavity of 2 cm^3 in volume and let $n = 1$.

(a) Convert this wavelength interval (1 Å) to frequency units (i.e., GHz and cm^{-1}).

(b) How many electromagnetic modes exist in this frequency band for this cavity?

(c) Suppose that the cavity were in the form of a cylinder with a cross-sectional area of 0.1 cm^2 (and thus is 20 cm long). How many $TEM_{0,0,q}$ cavity modes would fit within the frequency band specified by this 1 Å? (Do not forget the two polarizations.)

(d) Combine the results of (b) and (c) to estimate the probability of a spontaneous photon appearing in one of the polarized $TEM_{0,0,q}$ modes.

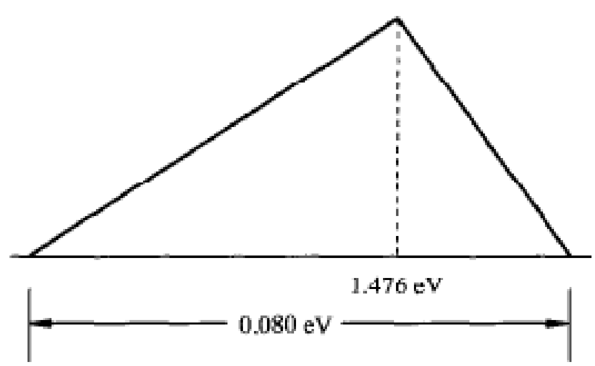
(e) If the A coefficient for this transition is 10^7 sec^{-1}, what is the stimulated emission cross section?

4

7.11. The spontaneous emission profile of a certain laser can be approximated by the triangular shape shown below. If the spontaneous lifetime were 5 nsec and the gain coefficient were 10 cm^{-1}, find

(a) The value of the line shape (in sec) at $h\nu/e = 1.476$ eV

(b) The inversion necessary to obtain that gain coefficient



Problem 1

$\tau\_spont := 1 \cdot 10^{-6} s$ $\quad \Delta\nu\_h := 1.5 \cdot 10^{12} Hz$ $\quad \lambda 0 := 5 \cdot 10^{-6} m$

$Nt := 2.5 \cdot 10^{19} cm^{-3}$ $\quad g1 := 5 \quad g2 := 1$ $\quad n := 1$

$$A21 := \frac{1}{\tau\_spont} \qquad g0 := \frac{1}{\Delta\nu\_h}$$

(a)

Cross Section: $\sigma 0 := A21 \cdot \frac{\lambda 0^2}{8 \cdot \pi \cdot n^2} \cdot g0$

$N2 := 0 \qquad N1 := Nt$

$$\gamma 0 := \sigma 0 \cdot \left(N2 - \frac{g2}{g1} \cdot N1 \right)$$

$\alpha 0 := -\gamma 0$

$$\alpha 0 = 3.316 \times 10^6 \frac{1}{m}$$

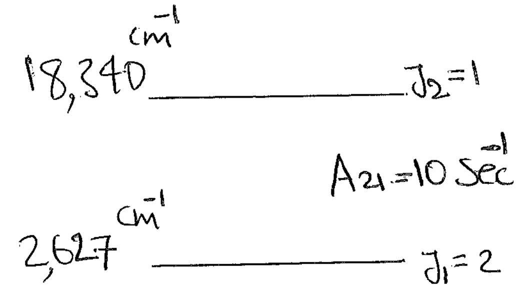
(b) For transparency (γ =0) we have FR*Nt-g2/g1 (1-FR)*Nt=0 where FR is the fraction of population that is excited to level 2.

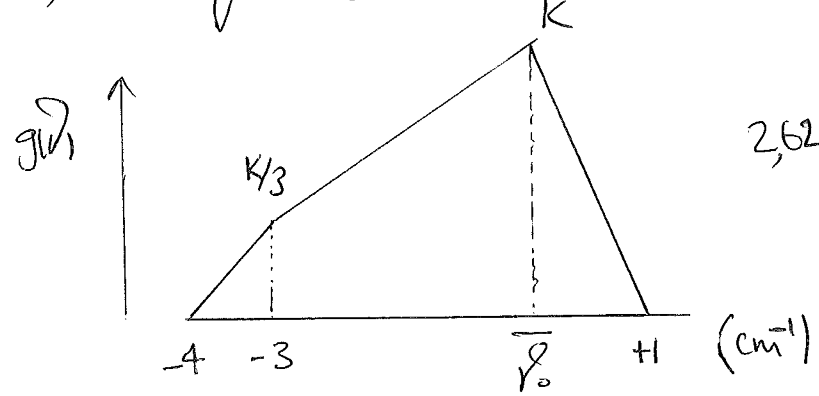
$$FR := \frac{\frac{g2}{g1}}{1 + \frac{g2}{g2}}$$

$FR = 0.1$ Fraction: 10 %

(5)

Problem (7.3) Verdeyen 3rd ed.





The value of $\bar{\nu}_0$ is $\bar{\nu}_0 = 18{,}343 - 2{,}627 = 15{,}719 \text{ cm}^{-1}$ and it is centered at zero in $g(\nu)$'s graph.

$$\int g(\nu)\,d\nu = 1 \Rightarrow \left(\frac{K}{3}\times\frac{1}{2} + (K/3 + K)\times\frac{3}{2} + \frac{K}{2}\right)\text{cm}^{-1} = 1$$

$$\Rightarrow \frac{8K}{3}\text{ cm}^{-1} = 1 \Rightarrow K = \frac{3}{8}\text{ cm} \qquad \left(\bar{\nu} = \frac{1}{\lambda}\right)$$

Now $\lambda = \frac{c}{\nu} \Rightarrow \nu = \frac{c}{\lambda} = \frac{3\times10^8}{10^{-2}\text{ m}} = 3\times10^{10}\text{ Hz}$

Thus $1\text{ cm}^{-1} = 3\times10^{10}\text{ Hz}$

$$\Rightarrow K = \frac{3}{8}\,\frac{1}{3\times10^{10}} = 1.25\times10^{-11}\text{ sec.} = g(\bar{\nu}_0)$$

We have: $\sigma(\nu) = A_{21}\,\frac{\lambda_0^2}{8\pi n^2}\,g(\nu)$

$$\lambda_0 = \frac{1}{15{,}719}\ \text{cm} = 636 \times 10^{-9}\ \text{m}$$

$$\Rightarrow \sigma_{SE}(\nu) = \frac{10\,(636 \times 10^{-9})^2}{8\pi} \times 1.25 \times 10^{-11} = 2.011 \times 10^{-24}\ \text{m}^2$$

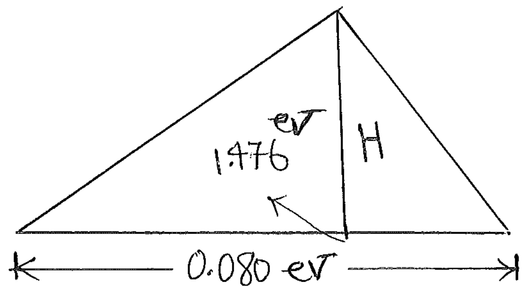
$$= 2.011 \times 10^{-20}\ \text{cm}^2$$

(b) using $g_{2(1)} = 2J_{2(1)} + 1 \Rightarrow g_1 = 5$ & $g_2 = 3$

Now $\sigma_{abs} = \frac{g_2}{g_1}\,\sigma_{SE}$ (Eq. 7.5.4)

$$\Rightarrow \sigma_{abs} = \frac{3}{5} \times 2.011 \times 10^{-20} = 1.2 \times 10^{-20}\ \text{cm}^2$$

Problem 7.11 Verdeyen 3rd ed.



(a) $0.080 \text{ eV} = h\Delta\nu \rightarrow \Delta\nu = \dfrac{0.08 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-34}} = 1.933 \times 10^{13} \text{ Hz}$

$1.476 \text{ eV} = h\nu_0 \longrightarrow \nu_0 = 3.56 \times 10^{14} \text{ Hz}$

$\int g(\nu)\, d\nu = 1 \Rightarrow \frac{1}{2}(1.933 \times 10^{13} \times H) = 1 \Rightarrow H = g(\nu_0) = 1.03 \times 10^{-13} \text{ sec.}$

(b) We know that : $\dfrac{dI_\nu}{dz} = \underbrace{\left(A_{21} \dfrac{\lambda_0^2}{8\pi n^2} g(\nu_0)\right) \overbrace{\left(N_2 - \dfrac{g_2}{g_1} N_1\right)}^{\Delta N}}_{\gamma(\nu) = 10/\text{cm}} I_\nu$

$\Rightarrow \qquad \lambda_0 = \dfrac{c}{\nu_0} = 842.7 \text{ nm}$

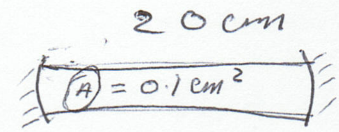
$\Rightarrow \Delta N = (10/\text{cm}) \dfrac{8\pi n^2}{A_{21} \lambda_0^2 g(\nu_0)} = 10^3 \dfrac{\times 8\pi \times 1}{\frac{1}{5 \times 10^{-9}} (842.7 \times 10^{-9})^2 \times 1.03 \times 10^{-13}}$

$\Rightarrow \Delta N = 1.72 \times 10^{21} / m^3$

7.5

$\lambda_0 = 5000\ \mathring{A}$ $\qquad \Delta\lambda = 1\ \mathring{A}$

$V = 2\ cm^3$



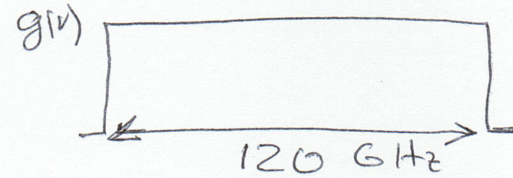
$\nu_0 = \frac{c}{\lambda_0} = 600\ THz$

(a) $\frac{\Delta\nu}{\nu} = \frac{\Delta\lambda}{\lambda} \Rightarrow \Delta\nu = 1.2 \times 10^{11}\ Hz$

$= 120\ GHz$

# of modes in volume V is (within $\Delta\nu$):

(b) $N = \frac{8\pi\nu^2\Delta\nu}{c^3} \times V \approx 8 \times 10^{10}$



$\Delta\nu_{FSR} = \frac{c}{2d} = 750\ MHz$

of $TEM_{0,0,q}$ modes in 120 GHz interval is:

(c) Polarization

$N_{TEM}\ \frac{120 \times 10^9}{750 \times 10^6} \times 2 = 320$

(d) Probability of spontaneous emission into one of $TEM_{0,0,q}$ is $\frac{320}{8 \times 10^{10}} \approx 4 \times 10^{-9}$ [small but possible]

(e) $\sigma = A_{21} \dfrac{\lambda^2}{8\pi n^2} g(\nu)$

$$g(\nu) \simeq \frac{1}{120\ \text{GHz}}$$

$A = 10^7$ $\qquad \lambda_0 = 0.5 \times 10^{-4}$ cm

$n = 1$

$$\sigma(\nu_0) = \frac{10^7 \times (0.5 \times 10^{-4})^2}{8\pi} \times \frac{1}{120 \times 10^9} \approx 8.3 \times 10^{-15}\ \text{cm}^2$$