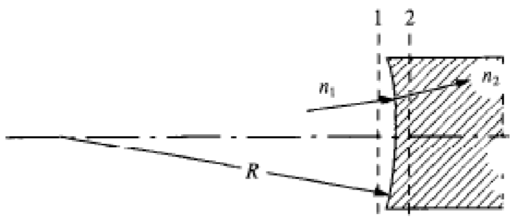
Laser Physics I (PHYS/ECE 464), Fall 2022

Homework #2, Due Monday, 9/12/22

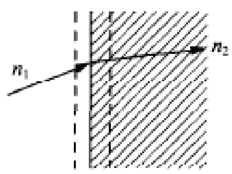
From *Verdeyen (3rd Edition)*: Problems 2.1, 2.2, 2.3, 2.4, 2.6

2.1. Derive the ray matrix for a ray entering a spherical dielectric interface.



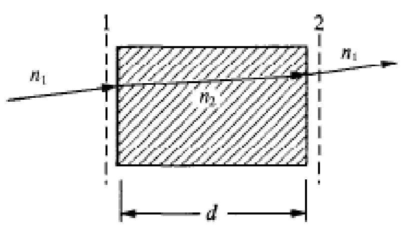
$$\text{Ans.: } \mathbf{T} = \begin{bmatrix} 1 & 0 \\ \left(1 - \frac{n_1}{n_2}\right)\frac{1}{R} & \frac{n_1}{n_2} \end{bmatrix}$$

2.2. Derive the ray matrix for the plane dielectric interface.



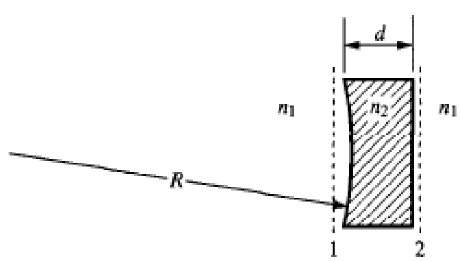
$$\text{Ans.: } \mathbf{T} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{bmatrix}$$

2.3. Derive the ray matrix for the plane dielectric slap of thickness d.

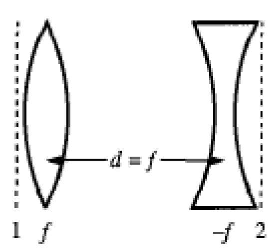


$$\text{Ans.: } \mathbf{T} = \begin{bmatrix} 1 & \frac{n_1 d}{n_2} \\ 0 & 1 \end{bmatrix}$$

2.4. Combine the results of problems 2.1 and 2.2 to derive the ray matrix for the negative lens. (Assume that $R \gg d$.)



2.6. Find the $ABCD$ matrix for the lens combination shown below.

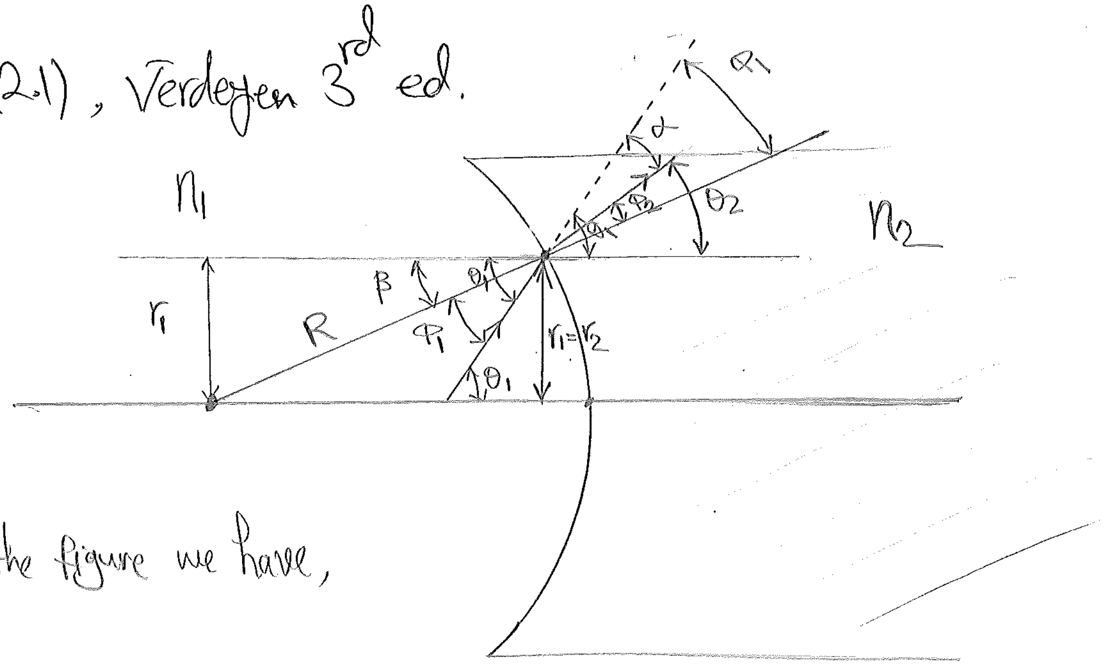


Laser Physics

Solutions of HW #2

1

Problem (2.1), Verdeyen 3^{rd} ed.



According to the figure we have,

$r_2 = r_1$

(paraxial optics; angles are small.) $r_1' = \theta_1 \simeq \tan\theta_1$; $r_2' = \theta_2 \simeq \tan\theta_2$

$\theta_2 = \theta_1 - \alpha$ on the other hand, $\alpha + \varphi_2 = \varphi_1 \Rightarrow \alpha = \varphi_1 - \varphi_2$

$\Rightarrow \theta_2 = \theta_1 - (\varphi_1 - \varphi_2)$; using Snell's law $n_1 \sin\varphi_1 = n_2 \sin\varphi_2 \Rightarrow n_1\varphi_1 \simeq n_2\varphi_2$

$\Rightarrow \theta_2 = \theta_1 - \varphi_1\left(1 - \frac{n_1}{n_2}\right)$

we can also write, $\varphi_1 + \beta = \theta_1 \Rightarrow \varphi_1 = \theta_1 - \beta \xrightarrow{\beta = \frac{r_1}{R}} \varphi_1 = \theta_1 - \frac{r_1}{R}$

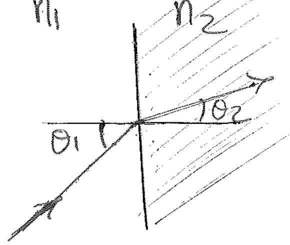
$$\Rightarrow \theta_2 = \theta_1 - \left(\theta_1 - \frac{r_1}{R}\right)\left(1 - \frac{n_1}{n_2}\right) = \cancel{\theta_1} - \cancel{\theta_1} + \frac{r_1}{R} + \frac{n_1}{n_2}\theta_1 - \frac{n_1}{n_2}\frac{r_1}{R}$$

$$= \left(1 - \frac{n_1}{n_2}\right)\frac{1}{R} r_1 + \frac{n_1}{n_2}\theta_1$$

$$\Rightarrow \begin{pmatrix} r_2 \\ r_2' \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 \\ \left(1 - \frac{n_1}{n_2}\right)\frac{1}{R} & \frac{n_1}{n_2} \end{pmatrix}}_{\equiv T} \begin{pmatrix} r_1 \\ r_1' \end{pmatrix}$$

θ_2 θ_1

② Problem (2.2) Verdeyen 3^{rd} ed.



$r_2 = 1 \times r_1 + 0 \times r_1'$

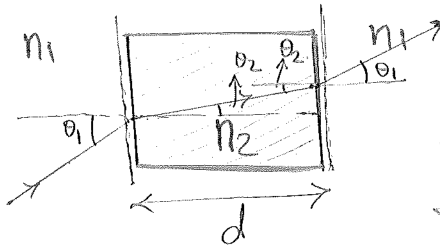
$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \xrightarrow[\text{optics}]{\text{Paraxial}} n_1\theta_1 = n_2\theta_2$$

$$\theta_2 = \frac{n_1}{n_2}\theta_1$$

$r_2' = 0 \times r_1 + \frac{n_1}{n_2} r_1'$

Thus $T = \begin{pmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{pmatrix}$

③ Problem (2.3) Verdeyen 3^{rd} ed.



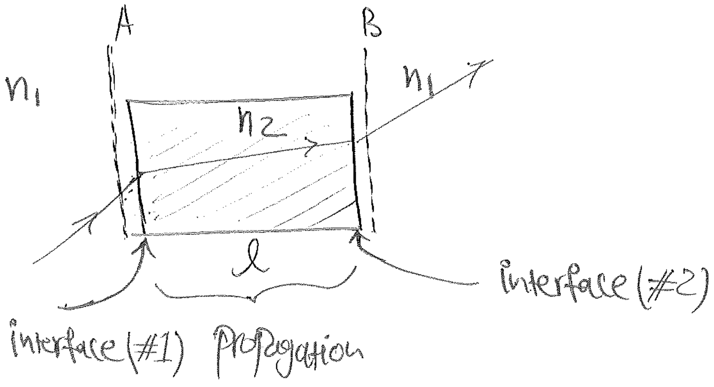
Approach 1.

$$\begin{cases} r_2 = r_1 + d\theta_2 = r_1 + d r_2' = r_1 + \frac{d n_1}{n_2} r_1' \\ r_2' = r_1' \end{cases}$$

$\Rightarrow$ Since with whatever angle the beam hits the slab at the left interface, exits the slab with the same angle at the right interface (Snell's law applies two times).

Thus, $T = \begin{pmatrix} 1 & \frac{d n_1}{n_2} \\ 0 & 1 \end{pmatrix}$

Approach 2.



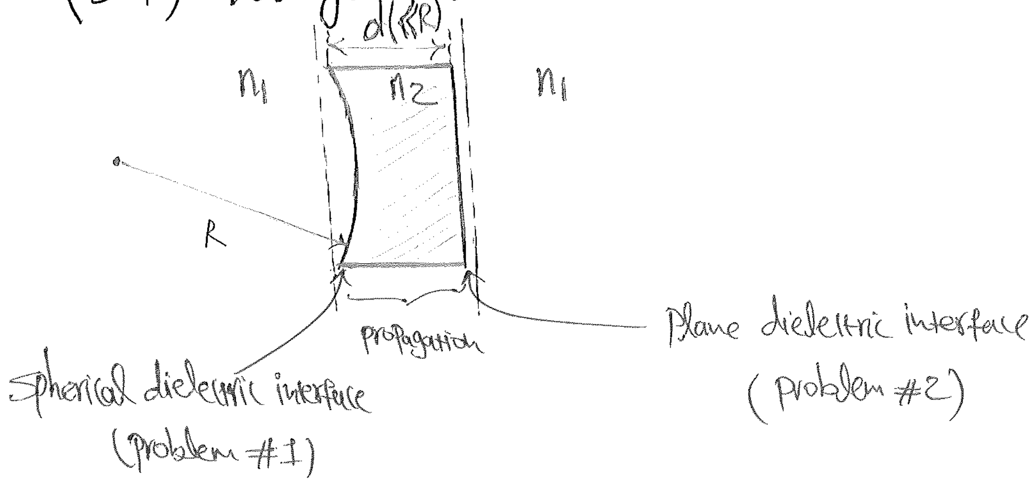
The matrix from A to B is simply the multiplication of three matrices, which all three are known.

$$T = \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & \frac{n_2}{n_1} \end{pmatrix}}_{\text{interface}(\#2)} \underbrace{\underbrace{\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}}_{\text{Propagation}} \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{pmatrix}}_{\text{interface}(\# 1)}}_{\begin{pmatrix} 1 & d\frac{n_1}{n_2} \\ 0 & \frac{n_1}{n_2} \end{pmatrix}}$$

$$\begin{pmatrix} 1 & d\frac{n_1}{n_2} \\ 0 & 1 \end{pmatrix}$$

Thus, $T = \begin{pmatrix} 1 & d\frac{n_1}{n_2} \\ 0 & 1 \end{pmatrix}$

④ Problem (24) Verdeyen 3rd ed.



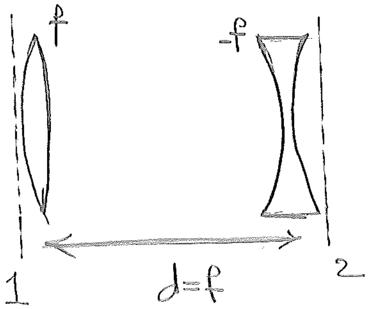
$$\text{Again, } T = \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & \frac{n_2}{n_1} \end{pmatrix}}_{\text{Plane interface}} \underbrace{\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}}_{\text{Propagation}} \underbrace{\begin{pmatrix} 1 & 0 \\ \frac{n_2-n_1}{n_2 R} & \frac{n_1}{n_2} \end{pmatrix}}_{\text{Spherical interface}}$$

$$\begin{pmatrix} 1+d\frac{(n_2-n_1)}{n_2 R} & d\frac{n_1}{n_2} \\ \frac{n_2-n_1}{n_2 R} & \frac{n_1}{n_2} \end{pmatrix}$$

$$\begin{pmatrix} 1+\frac{d(n_2-n_1)}{n_2 R} & d\frac{n_1}{n_2} \\ \frac{n_2}{n_1}\frac{(n_2-n_1)}{n_2 R} & 1 \end{pmatrix}$$

$$\Rightarrow T = \begin{pmatrix} 1+\frac{d}{R}\left(\frac{n_2-n_1}{n_2}\right) & d\frac{n_1}{n_2} \\ \frac{n_2-n_1}{n_1 R} & 1 \end{pmatrix}$$

(5) Problem (2.6) Verdeyen 3rd ed.



$$T = \underbrace{\begin{pmatrix} 1 & 0 \\ \frac{1}{f} & 1 \end{pmatrix}}_{\text{concave lens}} \underbrace{\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}}_{\text{propagation}} \underbrace{\begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}}_{\text{convex lens}}$$

$$\begin{pmatrix} 1-\frac{d}{f} & d \\ -\frac{1}{f} & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1-\frac{d}{f} & d \\ \frac{1}{f}\left(1-\frac{d}{f}\right)-\frac{1}{f} & \frac{d}{f}+1 \end{pmatrix}$$

Set $d=f$

$$T = \begin{pmatrix} 0 & f \\ -\frac{1}{f} & 2 \end{pmatrix}$$