Consider the double concave confocal cavity shown below:

(a) Find the roundtrip ABCD matrix (Choose the starting point to be just before mirror 2).
(b) Discuss the stability of this cavity for the symmetric $R_1=R_2$, and asymmetric ($R_1\neq R_2$) cases.
(c) A ray parallel to optical axis is incident on mirror 2 at a distance $x_0$ as shown. Derive an expression for the position $x(s)$ of this ray (on mirror 2) as a function of roundtrip number $s$. Discuss your results for cases $R_1=R_2$, $R_1>R_2$ and $R_1<R_2$.
(d) Draw the ray diagram for a few round trips in each case.
2.7) a -

$T = \left( \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right) \left( \begin{array}{cc} 1 & d/2 \\ 0 & 1 \end{array} \right) \left( \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right) \left( \begin{array}{cc} 1 & d/2 \\ 0 & 1 \end{array} \right)$

\[ T = \left( \begin{array}{cc} 1 & 3d \\ -d_f & -2d_f + 1 \end{array} \right) \]

Laser stability conditions:

\[ -1 < \frac{A + D}{2} < 1 \]

\[ -1 < \frac{1}{2} (1 - d_f - 2d_f + 1) < 1 \]

\[ -1 < \frac{3}{2} d_f + 1 < 1 \]

\[ -2 < -\frac{3}{2} d_f < 0 \quad 0 < d_f < \frac{4}{3} \]

2.8) a -

$b -

T = \left( \begin{array}{cc} 1 & d_1 \\ 0 & 1 \end{array} \right) \left( \begin{array}{cc} 1 & 0 \\ -\frac{1}{f_1} & 1 \end{array} \right) \left( \begin{array}{cc} 1 & d_2 \\ 0 & 1 \end{array} \right) \left( \begin{array}{cc} 1 & 0 \\ -\frac{1}{f_1} & 1 \end{array} \right) \left( \begin{array}{cc} 1 & d_1 \\ 0 & 1 \end{array} \right)$

\[ T = \left( \begin{array}{cc} 1 - \frac{2d_1}{f_1} - \frac{2d_2}{f_1^2} + \frac{2d_1 d_2}{f_1^2} & (1 - d_1)\left(1 - \frac{2d_2}{f_1} + 2d_1 d_2\right) - \frac{d_1}{f_1} + \frac{d_1^2}{f_1^2} \\ \frac{2d_2}{f_1^2} + \frac{2d_1 d_2}{f_1^2} & 1 - \frac{2d_1}{f_1} - \frac{2d_2}{f_1^2} + \frac{2d_1 d_2}{f_1^2} \end{array} \right) \]
\[ C \Rightarrow \left( \frac{A+D}{2} \right) \leq 1 \Rightarrow \left( 1 - \frac{2d_1}{f_1} - \frac{2d_2}{f_2} + \frac{2d_1d_2}{f_1^2} \right) < 1 \]

\[ 0 < \frac{d_1}{f_1} - \frac{d_2}{f_2} + \frac{d_1d_2}{f_1^2} < 1 \Rightarrow 0 \]

\[ 0 < \frac{d_1}{f_1} + \frac{d_2}{f_2} - \frac{d_1d_2}{f_1^2} < 1 \Rightarrow 0 \]

\[ 0 < \frac{1}{1 - \frac{d_1}{f_1}} \left( 1 - \frac{d_2}{f_2} \right) < 1 \]

Stable area.
3 Solution:

(a)\[\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} -R/R_2 & (R_1^2 - R_2^2)/R_2 R_1 \\ 0 & -R_2/R_1 \end{pmatrix}\]

(b) \[\frac{A + D}{2} = -\frac{1}{2} \left( \frac{R_1}{R_2} + \frac{R_2}{R_1} \right)\]

\((A + D)/2\) is always negative and thus satisfies \(<1\) condition, but it is also \(<-1\) for \(R_2 \neq R_1\). It is only marginally stable for \(R_1 = R_2\) where \(\frac{A + D}{2} = -1\).

(c) Starting from the ray position difference equation:

\[x^{s+2} - \frac{2(A + D)}{2} x^{s+1} + x^s = 0\]

Since the cavity is generally unstable, we seek a solution of the type \(x(s) = x_0 Z^s\) (i.e. not sinusoidal). You will then find the solutions to the resulting quadratic equation

\[Z^2 - \frac{2(A + D)}{2} Z + 1 = 0\]

to be:

\[Z_{1,2} = -\frac{R_1}{R_2}, \quad -\frac{R_2}{R_1} \]

Pick the one that satisfies the beam position (and or slope) in the first and second round trips (see also the procedure in the text), which gives:

\[x(s) = x_0 \left(-\frac{R_1}{R_2}\right)^s\]

For \(R_1 = R_2\), \(x(s) = x_0\); that is the ray returns to its original position in every round trip (i.e. marginally stable).

For \(R_1 > R_2\), ray position diverges and gets out of the cavity as \(s\) increase.
For \(R_1 < R_2\), ray position converges towards the center (axis) exponentially. You may alternatively write

\[x(s) = x_0 (-1)^s e^{-Ks}, \quad \text{where} \quad K = \ln \left(\frac{R_2}{R_1}\right).\]