Laser Physics I (PHYS/ECE 464), Fall 2022

Homework #9, Due Monday Nov. 14

1

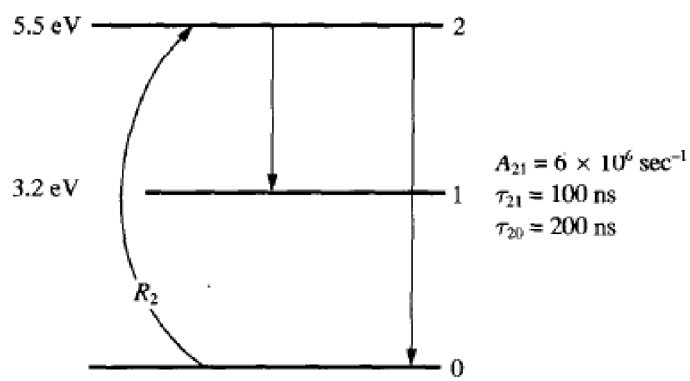
Consider the ideal laser medium shown below. The pump excites the atoms to state 2 at a rate R_2, which then decays to state 1 at a rate τ_{21}^{-1} and back to state 0 at a rate τ_{20}^{-1}. State 1 decays back to 0 so fast that the approximation $N_1 \approx 0$ is appropriate. The radiative rate for the 2 → 1 transition is 6×10^6 sec^{-1}, and its width is 10 GHz. (Assume a Lorentzian profile and steady state.)

(a) What is the stimulated emission cross section?

(b) What must be the pump rate R_2 in order to obtain a *small*-signal gain coefficient of 1%/cm?

(c) What is the saturation intensity for the 2 → 1 transition?

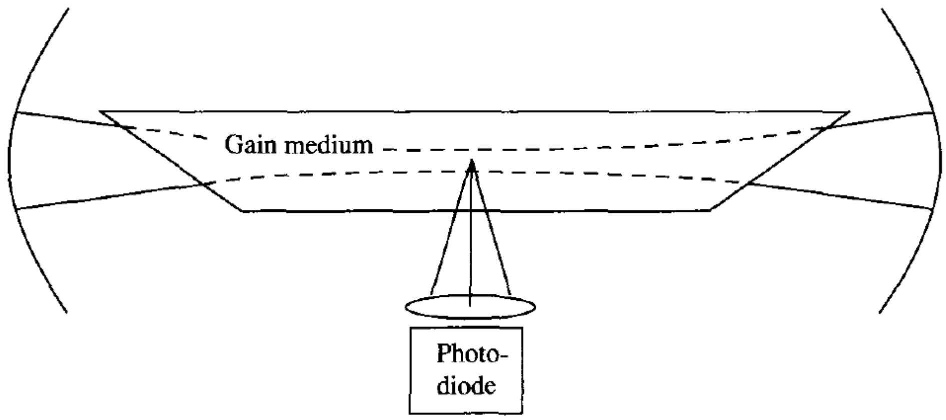
(d) How much power (in W/cm^3) is expended in creating the gain coefficient of (b)?



(e) Express the line width in Å units and cm^{-1} units.

2.

The purpose of this problem is to point out a simple experimental method for estimating the saturation intensity, $I_{sat.}$, of a laser. You are given the experimental apparatus shown below, which is made up of a continuously pumped gain medium (small-signal gain coefficient γ_0), two nearly perfect reflecting mirrors, and a photodetector. The photodetector records the side fluorescence power emanating from a small volume of the gain medium. Assume that the laser transition is homogeneously broadened and that the lower laser level population is negligible compared to that in the upper state.

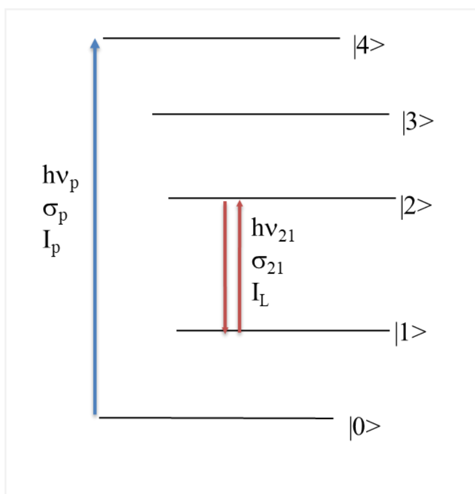


(a) If P_0 is the side fluorescence power (W/cm^{-3}) that is observed with one of the cavity mirrors blocked and P is measured when the laser is operating normally (i.e., mirror unblocked, everything else the same), then derive a simple expression that relates P/P_0 to the saturation intensity of the gain medium.

(b) If the side fluorescence is observed to be suppressed by 50% when the intercavity laser flux is 100 W/cm^2, what is $I_{sat.}$?

3. Rate Equations: Write down the rate equations for the following 5-level system where optical pumping is from the ground state to level 4, and stimulated absorption/emission is between levels 1 and 2.

The known paramers (in addtion to those shown in the figure) are each level liftime τ_j (j=1,2,3,4), branching rations ϕ_{ji} (i<j), and the total atomic density (N_t). Assume all degnerency factors are unity.



#1 (4) $\sigma(\nu_0) = \frac{A_{21}\lambda^2}{8\pi} g(\nu_0)$

$\Delta E = \frac{hc}{\lambda} \rightarrow \lambda = \frac{6.62\times10^{-34} \times 3\times10^{8}}{(5.5-3.2)(1.6\times10^{-19})} = 5.395\times10^{-7}$

for Lorantzian Profile: $g(\nu) = \frac{\Delta\nu}{2\pi\left[(\nu_0-\nu)^2+\left(\frac{\Delta\nu}{2}\right)^2\right]}\Bigg|_{\nu=\nu_0} = \frac{2}{\pi\Delta\nu}$

$\rightarrow g(\nu_0) = 636.94\times10^{-10}$.

$\rightarrow \sigma(\nu_0) = \frac{6\times10^{6} \times (5.34\times10^{-7})^2 \times (636.94\times10^{-10})}{8(3.14)} = 4.4\times10^{-14}\ cm^2$

(b) $\rightarrow$

$\gamma(\nu_0) = \sigma(\nu_0)\left[N_2 - N_1\right]$

$N_1 = 0$

$N_2 = R_2 \tau_2$ (S.S)

$\frac{1}{\tau_2} = \frac{1}{\tau_{21}} + \frac{1}{\tau_{20}}$

$\gamma(\nu_0) = \sigma(\nu_0) R_2 \left[\frac{1}{\tau_{21}} + \frac{1}{\tau_{20}}\right]^{-1}$

$0.01 = 4.4\times10^{-14} \times \left(\frac{1}{100\times10^{-9}} + \frac{1}{200\times10^{-9}}\right)^{-1} R_2$

$\rightarrow R_2 = 3.45\times10^{18} \left[\frac{cm^{-3}}{sec}\right]$

(c) $\rightarrow I_s = \frac{h\nu}{\sigma(\nu)\tau_2} = \frac{3.6\times10^{-19}}{4.4\times10^{-14}} \times \frac{3}{2}\times10^{7} = 122 \left[\frac{W}{cm^2}\right]$

(d) $\rightarrow \frac{\Delta P}{\text{area}\cdot\text{length}} = \frac{\gamma_0(\nu) I_s}{g(\nu)} = \sigma_0(\nu) R_2 \tau_2 \frac{h\nu}{\sigma_0(\nu)\tau_2}$

$= R_2 h\nu = R_2 \Delta E = 3.45\times10^{18} \times 3.6\times10^{-19}$

$= 12.69\times10^{-1} = 1.269 \left[\frac{W}{cm^3}\right]$

(e) $\rightarrow \Delta\nu = 10^{10}$ Hz

$\nu = \frac{\Delta E}{h} = \frac{(5.5-3.2)\times(1.6\times10^{-19})}{6.62\times10^{-34}} = 0.55\times10^{15}$

5.

$\rightarrow \Delta\lambda = \frac{c\Delta\nu}{\nu^2} = 10^{-11}\ m = 0.1\ A^\circ$

#2

(a) For this problem, the key is to recall that when the laser is oscillating, the upper state population N_2, and thus the gain saturates to loss levels. When the cavity is blocked, N_2 increases back to its unsaturated (small signal) value.

Thus

$$N_2 \approx \frac{N_2^0}{1 + I_\nu / I_s}$$

where I_ν is the total intracavity intensity.

The side fluorescence (spont. Emission) power (from a gain volume V) is

$$P = A_{21} N_2 h\nu V$$

The ratio of fluorescence power in two cases of lasing on and off is therefore:

$$\frac{P}{P_0} = \frac{N_2}{N_2^0} = \frac{1}{1 + I_\nu / I_s}$$

(b) $\frac{P}{P_0} = \frac{1}{2}$ leading to $\frac{I_\nu}{I_s} = 1$

(Also remember in a high-Q cavity, output intensity would have been $I_{out} \approx \frac{T_2 I_\nu}{2}$, in case one was interested!).

#3

$$\frac{dN_4}{dt} = \frac{\sigma_p (N_0 - N_4)}{h\nu_p} I_p - \frac{N_4}{\tau_4}$$

$$\frac{dN_3}{dt} = \Phi_{43} \frac{N_4}{\tau_4} - \frac{N_3}{\tau_3}.$$

$$\frac{dN_2}{dt} = \Phi_{42} \frac{N_4}{\tau_4} + \Phi_{32} \frac{N_3}{\tau_3} - \frac{N_2}{\tau_2} - \frac{\sigma_{21} (N_2 - N_1)}{h\nu_{12}} I_L.$$

$$\frac{dN_1}{dt} = \Phi_{41} \frac{N_4}{\tau_4} + \Phi_{31} \frac{N_3}{\tau_3} + \Phi_{21} \frac{N_2}{\tau_2} - \frac{N_1}{\tau_1} + \frac{\sigma_{21} (N_2 - N_1) I_L}{h\nu_{12}}.$$

$$N_0 + N_1 + N_2 + N_3 + N_4 = N_{Total} \equiv \text{Constant}$$