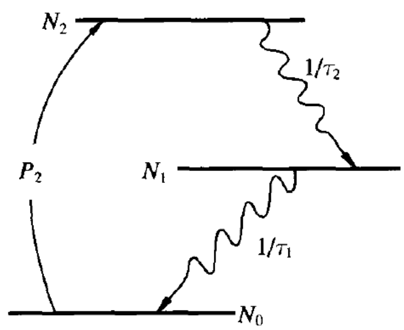
Laser Physics I (PHYS/ECE 464), Fall 2022

Homework #8, Due Monday Nov. 7

1.

In the diagram shown below, the pump P_2 (i.e., electrons, flash lamp, another laser, etc.) excites atoms from state 0 to state 2, nothing to state 1. To make the problem simple and tractable, assume state 0 is not depleted to any significant extent for any time (i.e., $dN_0/dt = 0$); use the simple decay route indicated; neglect stimulated emission; assume $\tau_2 = 1\ \mu s$ and $\tau_1 = 2\ \mu s$; and let $P_2 = 10^{20}\ cm^{-3}\ s^{-1}$. Use symbols for (a) and (b) and find numerical values for (c) and (d).



(a) What are the rate equations for states 2 and 1?
(b) Give an expression for the densities $N_{2,1}$ as a function of time.
(c) Over what time interval, δt, is the population difference $N_2 - N_1 > 0$?
(d) What are the steady-state populations in 2 and 1?

2.

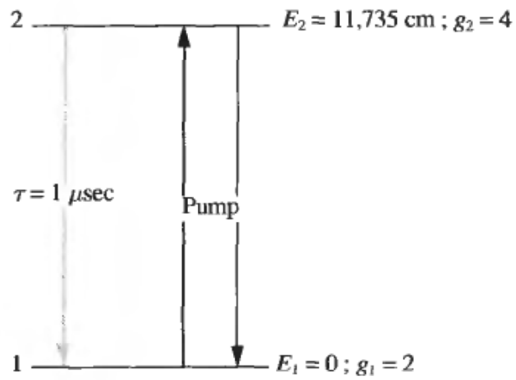
Consider the atomic system shown below being irradiated by an external wave tuned to the center of the $2 \rightarrow 1$ transition with 1 being the ground state. The wave pumps the atoms from 1 to 2 and also stimulates the atoms back to 1 from 2. In addition, the atoms in state 2 decay back to 1 by spontaneous emission and/or by other processes with a rate given by $(\tau)^{-1}$. The total density of atoms is $[N]$. Assume $\sigma = 10^{-14}\ cm^2$

(a) Formulate the rate equations for the two states in terms of the intensity of the external wave, the stimulated emission cross section, the frequency $h\nu = E_2 - E_1$, τ, and the degeneracies of the states (g_2, g_1).

(b) What would be the population *ratio* N_2/N_1 if the intensity of the external wave were infinite?

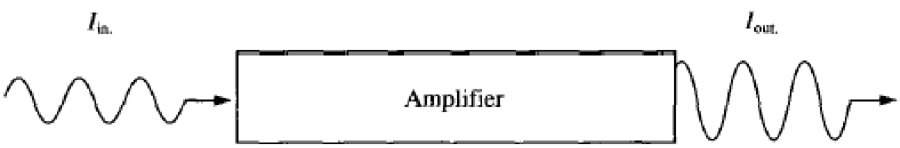
(c) What must be the intensity to make the population ratio N_2/N_1 equal to $1/2$?

(d) If the ambient temperature were such that $kT = 208\ \text{cm}^{-1}$ and the intensity were zero, what is the steady state population ratio N_2/N_1?



3.

An experiment involving a homogeneously broadened optical amplifier is depicted in the diagram below. For an input intensity of $1\ W/\text{cm}^2$, the gain (output/input) is 10 dB. If the input intensity is doubled to $2\ W/\text{cm}^2$, the gain is reduced to 9 dB.



(a) What is the small-signal gain (i.e., $I_{in} \to 0$) of this amplifier (in dB)?

(b) What is the saturation intensity?

(c) What is the maximum power (per unit area) that can be extracted from this amplifier (in limit of large input intensity)?

(d) What must be the input intensity to extract 50% of this maximum?

Note: dB (gain)= 10 $\log_{10}(P_{out}/P_{in})$ or 10 $\log_{10}(I_{out}/I_{in})$

1.

(a) $\frac{dN_2}{dt} = \frac{\sigma I_p}{h\nu}\left[\frac{g_2}{g_1} N_1 - N_2\right] - \frac{N_2}{\tau}$; There is no need to write a separate equation for N_1 since atoms must be conserved and thus $N_1 + N_2 = [N]$;

(b) If $I_p \to \infty$, then the bracket in the above must be zero to prevent an infinity appearing in a physical equation. Thus $N_2/N_1 = g_2/g_1$.

(c) For a steady-state situation: $N_2\left[1 + \frac{\sigma\tau}{h\nu} I_p\right] = \frac{g_2}{g_1}\frac{\sigma\tau}{h\nu} I_p N_1$ or $\frac{N_2}{N_1} = \frac{(g_2/g_1)\cdot(I_p/I_s)}{1 + (I_p/I_s)}$

where $I_s = \frac{h\nu}{\sigma\tau}$; This ratio equals 0.5 when $I_p/I_s = 0.25$ for $g_2/g_1=2$; $I_s = 23.3$ W/cm^2;

Thus $I = 5.82$W/cm^2; (d) $N_2/N_1 = (g_2/g_1)\exp[-E/kT] = 6.25\times10^{-25}$; safe to ignore N_2.

2.

This is a reasonable model for the pulsed N_2 laser which has a very high gain and lases at 0.337 μm. Its major drawback is that it is self-terminating because the inversion can not be maintained indefinitely – fact that comes out of the analysis that follows.

$\frac{dN_2}{dt} = P_2 - \frac{N_2}{\tau_2}$ whose solution is: $N_2(t) = P_2\tau_2(1 - \exp[-t/\tau_2])$

$\frac{dN_1}{dt} = +\frac{N_2}{\tau_2} - \frac{N_1}{\tau_1}$ or $\frac{dN_1}{dt} + \frac{N_1}{\tau_1} = P_2(1 - \exp[-t/\tau_2])$

$$N_1 = P_2\tau_1\left\{1 - \frac{\tau_1/\tau_2}{\tau_1/\tau_2 - 1}e^{-t/\tau_1} + \frac{1}{\tau_1/\tau_2 - 1}e^{-t/\tau_2}\right\}$$

It is informative to consider the steady-state populations, i.e., where $t >> \tau_{2,1}$. $N_2(t\to\infty) = P_2\tau_2$ and $N_1(t\to\infty) = P_2\tau_1$. Note that since $\tau_1 > \tau_2$, we have the undesirable situation of $N_1 > N_2$ which means that the system will not lase in a steady state. However, one can obtain a transient inversion (and a laser) as the sketch below indicates.

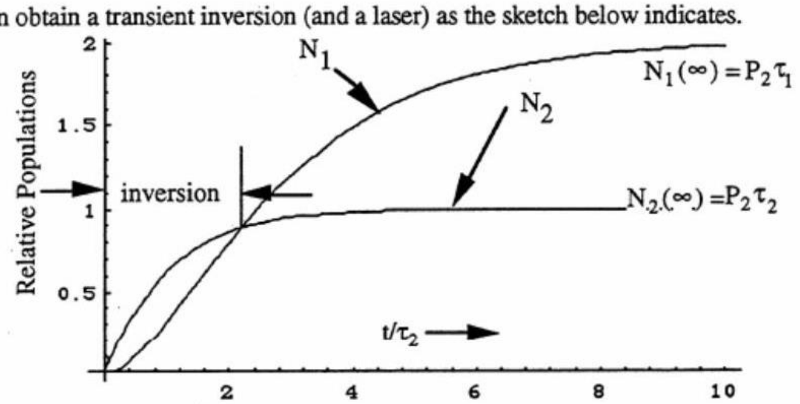


Figure for problem 7.10

$N_2(t\to\infty) = 10^{20} \times 10^{-6} = 10^{14}$ cm^{-3}, $N_1(t\to\infty) = 10^{20} \times 2 \times 10^{-6} = 2 \times 10^{14}$ cm^{-3}

Let $t/\tau_1 = x$; $(\tau_1/\tau_2) = 2$; $N_1 = N_2$ when $2(1 - 2e^{-x} + e^{-2x}) = (1 - e^{-2x})$;

Collect terms, multiply by e^{2x} and factor: $f(x) = [e^x - 3][e^x - 1] = 0$;

where $x = t/\tau_1$; $F(x) = 0$ at $x = 0$ (i.e.at the start) or $x = t/\tau_1 = \ln 3$; $\therefore$ $t = 1.0986\tau_1 = 2.2$ μsec

3

$$I_{out} = 10^{\frac{G_{dB}}{10}} I_{in} \rightarrow \begin{cases} \text{for } G = 10 \text{ dB} \rightarrow \frac{I_{out}}{I_{in}} = 10 = G_1 \\ \text{for } G = 9 \text{ dB} \rightarrow \frac{I_{out}}{I_{in}} = 10^{0.9} \simeq 7.9 = G_2 \end{cases}$$

$$\ln G + \bar{g}(\nu) \frac{I_{in}}{I_s}(G-1) = \gamma_0 \cdot l_g \quad ; \quad \bar{g}(\nu) = 1$$

$$\begin{cases} \ln G_1 + \frac{I_{in}}{I_s}(G_1 - 1) = \gamma_0 l_g \\ \ln G_2 + \frac{I_{in}}{I_s}(G_2 - 1) = \gamma_0 l_g \end{cases} \quad \begin{cases} \ln 10 + \frac{1}{I_s}(10-1) = \gamma_0 l_g \\ \ln 7.9 + \frac{2}{I_s}(7.9-1) = \gamma_0 l_g \end{cases}$$

$$\rightarrow \begin{cases} 2.3 + \frac{9}{I_s} = \gamma_0 l_g \\ 2.06 + \frac{13.8}{I_s} = \gamma_0 l_g \end{cases} \rightarrow \begin{cases} \gamma_0 l_g = 2.75 \\ I_s = 20 \left[\frac{W}{cm^2}\right] \end{cases} \quad (b)$$

$$\rightarrow G_0 = \exp(\gamma_0 l_g) = 15.64 = 11.94 \text{ dB} \quad (a)$$

(c) $$\frac{\Delta P}{\text{area} \cdot \text{length}} = \frac{\gamma_0(\nu) I_s}{\bar{g}(\nu)} \rightarrow \frac{\Delta P}{\text{area}} = l_g \gamma_0(\nu) I_s$$

$$= (2.75) \cdot 20 = 55 \left[\frac{W}{cm^2}\right]$$

(d) $$\begin{cases} I_{out} = I_{in} + \left[\frac{\gamma_0(\nu) I_s l_g}{\bar{g}(\nu)}\right] \\ I_{out} = \frac{1}{2} I_{in} \end{cases} \nearrow \quad I_{out} - I_{in} = \frac{1}{2} \times 55 = 27.5$$

$$\rightarrow \ln G + \frac{I_{in}}{I_s}(G-1) = \gamma_0(\nu) l_g \rightarrow \ln \frac{I_{out}}{I_{in}} + \frac{I_{in}}{I_s}\left(\frac{I_{out}}{I_{in}} - 1\right) = \gamma_0 l_g$$

$$\rightarrow \ln \frac{I_{out}}{I_{in}} + \frac{I_{in}}{20}\left(\frac{I_{out} - I_{in}}{I_{in}}\right) = 2.75 \rightarrow$$

$$\ln \frac{I_{out}}{I_{in}} + \frac{27.5}{20} = 2.75 \rightarrow \begin{cases} I_{out}/I_{in} = 3.93 \\ I_{out} - I_{in} = 27.5 \end{cases} \rightarrow I_{in} = 9.38 \left[\frac{W}{cm^2}\right]$$