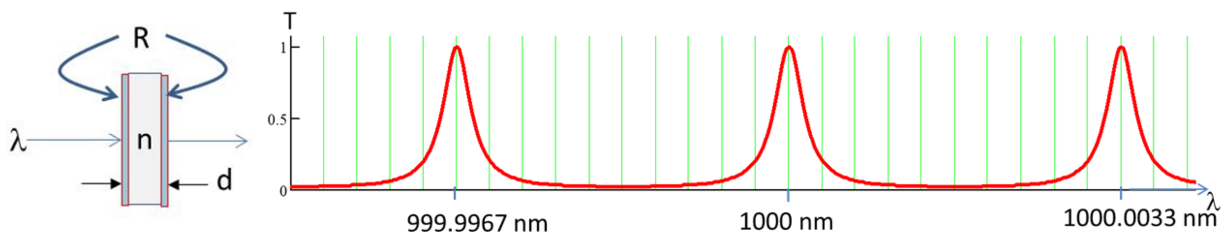
Laser Physics-I (PHYC/ECE 464), Fall 2022

Homework #6, Due Monday Oct. 10

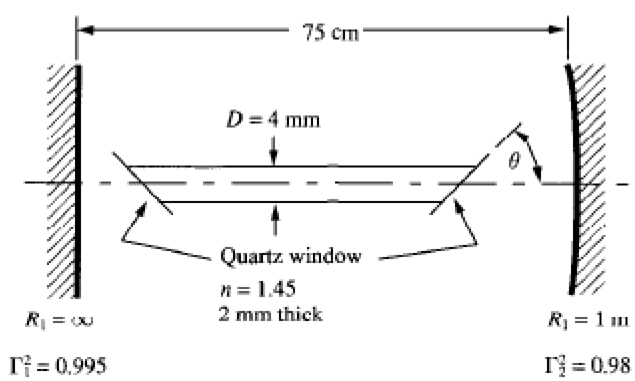
1- Drawn to scale on the graph below is the relative power transmission of a tunable light source at normal incidence through a Fabry-Perot etalon as the wavelength λ is varied. The etalon is made from of glass having index n=1.515 and thickness d with both sides mirrored with reflectivity *R*.

a. What is *d*?
b. What is the *Finesse* , Q, and the reflectivity *R*?
c. Draw (on top of the above graph) the transmission for the case where *R* is purely due to the Fresnel reflectivities at normal incidence (i.e. no coating). What is the *finesse* and the *minimum transmission* in this case?



2. Problem 6.21 from Verdeyen:

Consider the accompanying diagram of a cavity designed to be utilized with a helium/neon laser at $\lambda_0 = 632.8$ nm.

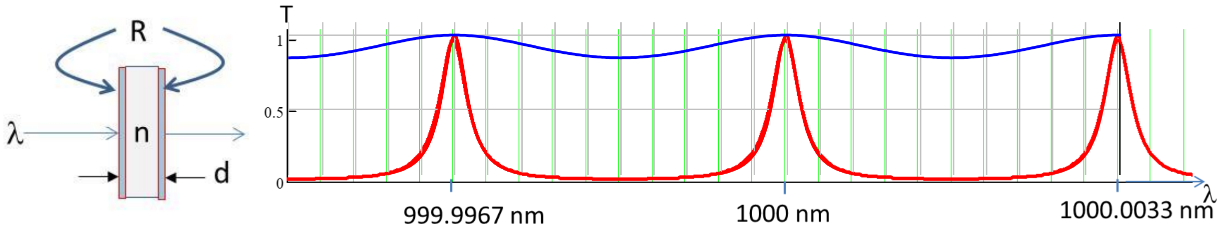


(a) Is the cavity stable?
(b) What is the spot size of the beam at the flat mirror?
(c) What is the spot size of the beam at the spherical mirror?
(d) The windows are cemented to the tube at Brewster's angle. What is the angle θ as shown on the sketch?
(e) Assuming that the tube bore is centered with respect to the axis of the $TEM_{0,0}$ mode, compute the loss introduced by the aperturing action of the tube walls. (Zero is *not* an acceptable answer.)
(f) What is the formula for the resonant frequency of the $TEM_{m,p,q}$ mode?

3. **Problem 6.22 from Verdeyen:**

Consider a single $TEM_{0,0,q}$ mode of the laser shown in Problem 6.21. Because of room vibrations, sound waves, and temperature variations, the distance $d = 75$ cm varies slightly about its nominal value. If the optical frequency of a mode is to be held constant to 1 kHz, what is the maximum allowable variation in d? The answer should disturb you, especially when you consider that atoms are *spaced* about 4 Å apart. Nevertheless, such frequency control is possible.

Drawn to scale on the graph below is the relative power transmission of a tunable light source at normal incidence through a Fabry-Perot etalon as the wavelength λ is varied. The etalon is made from of glass having index $n=1.515$ and thickness d with both sides mirrored with reflectivity R.



a. What is d? (8 points)

$$\Delta\lambda FSR := (1000 - 999.9967)\cdot 10^{-9}\,\text{m} \qquad \frac{\Delta\nu FSR}{\nu} := \frac{\Delta\lambda FSR}{\lambda} \qquad \nu := \frac{c}{\lambda}$$

$$\Delta\nu FSR := \frac{c}{\lambda^2}\cdot\Delta\lambda FSR$$

$$\Delta\nu FSR = 9.893\times 10^{8}\,\frac{1}{s}$$

$$d := \frac{c}{2\cdot n\cdot\Delta\nu FSR}$$

$$d = 1\times 10^{-1}\,\text{m}$$

b. What is the *estimated* *Finesse* and the reflectivity R? (7 points)

$$\Delta\nu 12 := \Delta\nu FSR\cdot\frac{1}{10}$$

$$\text{Finesse} := \frac{\Delta\nu FSR}{\Delta\nu 12}$$

$$\text{Finesse} = 1\times 10^{1} \qquad F := \text{Finesse}$$

$$R := 1 + \frac{\pi^2}{2\cdot F^2} - \sqrt{\left(1 + \frac{\pi^2}{2\cdot F^2}\right)^2 - 1}$$

$$R = 7.313\times 10^{-1}$$

c. Draw (on top of the above graph) the transmission for the case where R is purely due to the Fresnel reflectivities at normal incidence (i.e. no coating). What is the *finesse* and the *minimum transmission* in this case? (10 points)

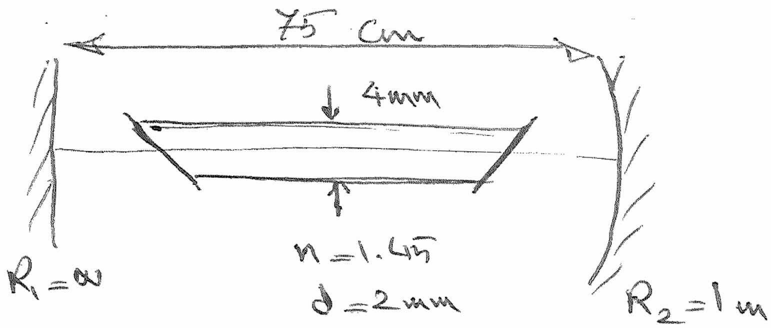
(c)

$$R := \left(\frac{n-1}{n+1}\right)^2$$

$$F := \frac{\pi\sqrt{R}}{1-R} \qquad F = 6.715\times 10^{-1}$$

$$\text{Tmin} := \frac{1}{1 + \left(\frac{2}{\pi}F\right)^2}$$

$$\text{Tmin} = 8.455\times 10^{-1}$$



$$\begin{cases} g_1 = 1 \\ g_2 = 1 - \dfrac{75}{100} = 0.25 \end{cases}$$

(a) $g_1 g_2 = 0.25 < 1 \rightarrow$ Stable.

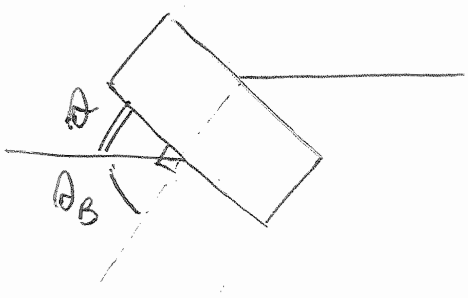
(b) $z_0 = \sqrt{d R_2} \left(1 - \dfrac{d}{R_2}\right)^{1/2} = 43.2$ cm.

$\dfrac{\pi w_0^2}{\lambda_0} = 43.2 \rightarrow w_0 = 295\ \mu m$ (at flat mirror)

(c) $w(z = 75) = w_0 \left(1 + \dfrac{z^2}{z_0^2}\right)^{1/2}$

$\rightarrow W = 591\ \mu m$ (at the curved mirror)

(d)



$$\begin{cases} \theta = \dfrac{\pi}{2} - \theta_B \\ \theta_B = \tan^{-1} n = \tan^{-1} 1.45 \end{cases}$$

$\rightarrow \theta = 34.6^\circ$

(e) Loss is Max at the Curved mirror end when W is largest.

$$\frac{D}{2W} = \frac{2\,mm}{0.591}$$

$$loss = 1 - \frac{\int_0^{D/2} e^{-\frac{2r^2}{w^2}}\, r\,dr}{\int_0^{\infty} e^{-\frac{2r^2}{w^2}}\, r\,dr} = e^{-\frac{2D^2}{4w^2}} = 1.13\times10^{-10}$$

(very small)

(f) $$\nu_{m,p,q} = \frac{c}{2d}\left\{ q + \frac{(1+m+p)}{\pi} \tan^{-1}\frac{d}{z_0} \right\}$$

(ignore the effects of windows)

6.22

$\nu = q \frac{c}{2nd} \rightarrow \Delta\nu = \frac{qc}{2n}\left(-\frac{\Delta d}{d^2}\right) \rightarrow$

$\Delta\nu = \frac{qc}{2nd}\left(\frac{\Delta d}{d}\right) \rightarrow \frac{\Delta\nu}{\nu} = \frac{\Delta d}{d}$

$\lambda_0 = 632 \times 10^{-9}\ m \rightarrow$

$\Delta d = \frac{\Delta\nu \cdot d}{c/\lambda_0} = \frac{(0.75)(1\times10^{3})(632.8\times10^{-9})}{3\times10^{8}} = 1.58 \times 10^{-12}\ m$