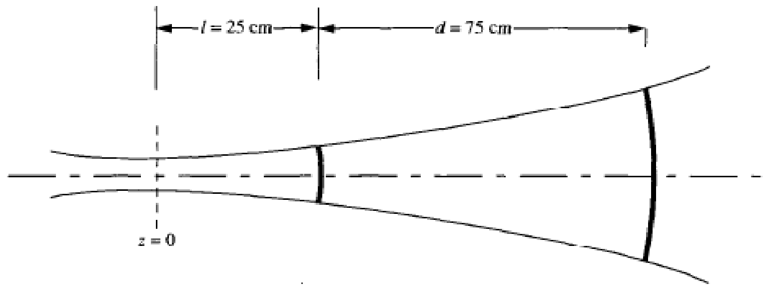
Laser Physics-I (PHYC/ECE 464), Fall 2022

Homework #5, Due Monday Oct. 3

1

In the stable optical cavity shown in the diagram below, the plane $z = 0$ occurs at a distance 25 cm to the left of M_1 with the beam parameter $z_0 = 125$ cm. The distance between the two mirrors is 75 cm.

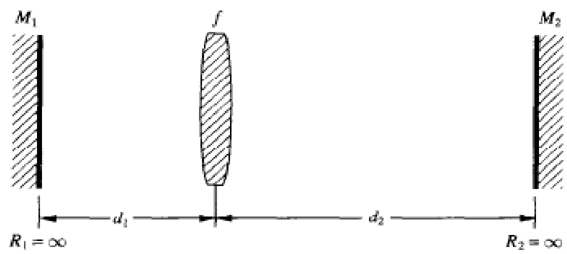


(a) Find a formula for the resonant frequency of the $\mathrm{TEM}_{m,p,q}$ mode.
(b) Find the difference between the resonance frequency of the $\mathrm{TEM}_{1,2,q}$ and $\mathrm{TEM}_{0,0,q}$ modes.
(c) Find the radius of curvature for the mirrors M_1 and M_2.

2

Consider the optical cavity consisting of two flat mirrors with a converging lens as shown in the accompanying diagram.

(a) What are the stability limits for this cavity? Express your answer in the form of an inequality involving the ratio of d_1/f and d_2/f.
(b) Construct a stability diagram expressing this inequality.

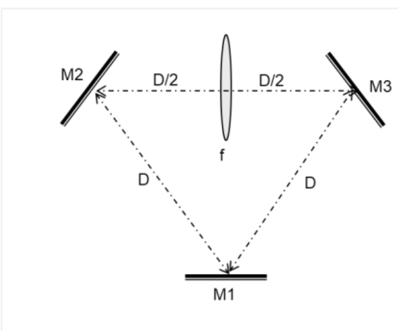


3

Find the spot sizes at the mirrors M_1 and M_2 of the cavity shown in Problem **2**

4.

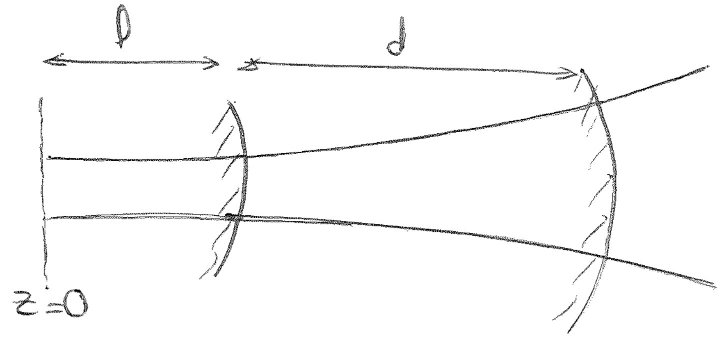
Consider the ring laser cavity shown in the accompanying diagram.



(a) Show an equivalent-lens waveguide for this cavity and identify a unit cell starting at mirror 1.
(b) What is the transmission matrix for this unit cell?
(c) What are the values of D/f that make this a stable cavity?
(d) Where is the location (z=0) of minimum beam waist (w_0)? Explain.
(e) Obtain w_0 in terms of given parameters and λ_0.

Note: Problems 2 and 4 have much in common with those in HW#3. So, either copy or just skip the common parts.

5.6



$\varphi_1 = \varphi(z=0 \text{ to } l) = kl - (1+m+p)\tan^{-1} l/z_0$

$\varphi_2 = \varphi(z=0 \text{ to } d+l) = k(l+d) - (1+m+p)\tan^{-1}\dfrac{l+d}{z_0}$

$$\Phi(z=l \to l+d) = \varphi_2 - \varphi_1 = kd - (1+m+p)\left[\tan^{-1}\frac{l+d}{z_0} - \tan^{-1}\frac{l}{z_0}\right]$$
$$= q\pi$$

(a) $$\nu_{m,p,q} = \frac{c}{2d}\left\{q + \frac{(1+m+p)}{\pi}\left[\tan^{-1}\frac{l+d}{z_0} - \tan^{-1}\frac{l}{z_0}\right]\right\}$$

$$\nu_{1,2,q} - \nu_{0,0,q} = \frac{c}{2d}\left[\left(\frac{4}{\pi} - \frac{1}{\pi}\right)\left(\tan^{-1}\frac{l+d}{z_0} - \tan^{-1}\frac{l}{z_0}\right)\right]$$

$$= \frac{c}{2d}\cdot\frac{3}{\pi}\left[\tan^{-1}\frac{100}{125} - \tan^{-1}\frac{25}{125}\right]$$

$$= 200\text{ MHz} \times \frac{3}{\pi}\left[\tan^{-1}\frac{100}{125} - \tan^{-1}\frac{25}{125}\right]$$

(b) $\Rightarrow \Delta\nu = 91.16$ MHz.

(c) $$-R_1 = \left.\frac{z^2 + z_0^2}{z}\right|_{z=l} = \frac{l^2 + z_0^2}{l} = 650 \text{ cm}.$$

$\to R_1 = -650$ cm

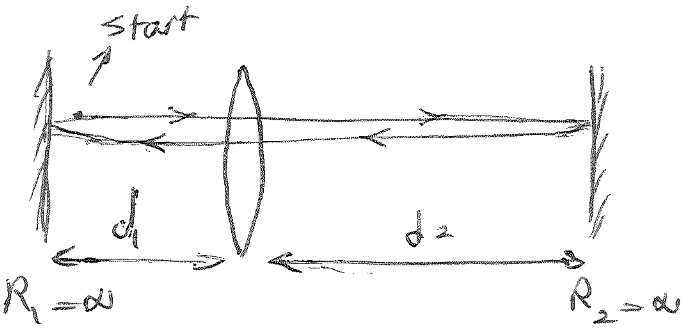
$$R_2 = \frac{(l+d)^2 + z_0^2}{l+d} = 265.$$

$\to R_2 = 265.$

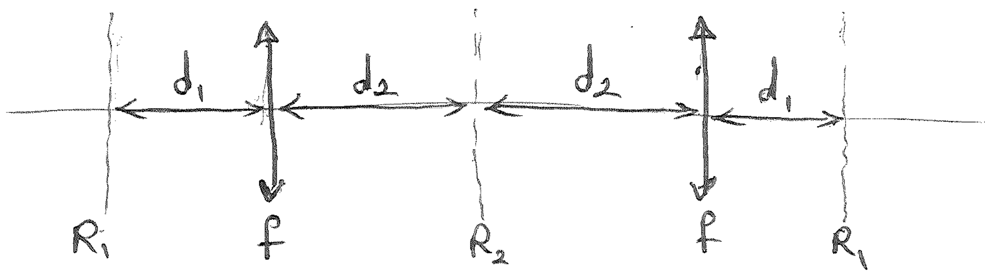
(5.7) & (5.8)

we will follow the recipe to get the problem solved.

5.7) i) Find the ABCD Matrix of a round trip.



unit cell:



$$T = \begin{pmatrix} 1 & d_1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \begin{pmatrix} 1 & 2d_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \begin{pmatrix} 1 & d_1 \\ 0 & 1 \end{pmatrix}$$

$$T = \begin{pmatrix} 1 - \frac{2d_2}{f} - \frac{2d_1}{f}\left(1 - \frac{d_2}{f}\right) & d_1 + 2d_2\left(1 - \frac{d_1}{f}\right) + d_1\left(1 - \frac{2d_1}{f} - \frac{2d_2}{f}\left(1 - \frac{d_1}{f}\right)\right) \\ \frac{-2}{f}\left(1 - \frac{d_2}{f}\right) & 1 - \frac{2d_1}{f} - \frac{2d_2}{f}\left(1 - \frac{d_1}{f}\right) \end{pmatrix}$$

$$\left(1-\left(\frac{A+D}{2}\right)^2\right) > 0$$

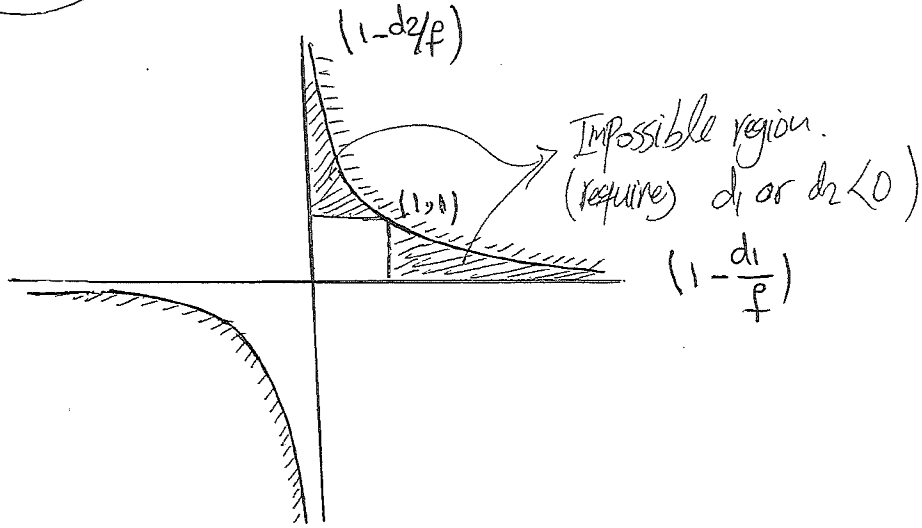
$$\Downarrow$$

Stability: $-1 \leq \frac{A+D}{2} \leq 1 \rightsquigarrow -2 \leq A+D \leq 2$

$$-2 \leq 2 - \frac{2d_2}{f}\left(2-\frac{d_1}{f}\right) - \frac{2d_1}{f}\left(2-\frac{d_2}{f}\right) \leq 2$$

$$\Rightarrow 0 \leq 2 - \frac{2d_1}{f}\left(1-\frac{d_2}{f}\right) - \frac{2d_2}{f} \leq 2$$

$$\Rightarrow 0 \leq \left(1-\frac{d_1}{f}\right)\left(1-\frac{d_2}{f}\right) \leq 1$$



5-8 We can easily get the spot size on mirror M_1 since we have obtained the ABCD matrix for that.

$$R\Big|_{M_1\,(\text{or } z=0)} = \underbrace{\frac{-2B}{A-D}}_{\text{Eq.}(5.3.6)} \to \infty \Longrightarrow A = D$$

$$\frac{\pi n W^2(z)}{\lambda_0} \overset{\text{Eq.}(5.3.7)}{=} \frac{B}{\left(1-\left(\frac{A+D}{2}\right)^2\right)^{1/2}} = \frac{B}{(1-D^2)^{1/2}}$$

$$\Rightarrow \frac{\pi n W_{01}^2}{\lambda_0} = \frac{\overbrace{d_1 + 2d_2(1 - d_1/f) + d_1\left(1 - 2d_1/f - 2d_2/f(1 - d_1/f)\right)}^{\alpha}}{\left(1 - \left(1 - \frac{2d_1}{f} - \frac{2d_2}{f}\left(1 - \frac{d_1}{f}\right)\right)^2\right)^{1/2}}$$

$$\Rightarrow \frac{\pi n W_{01}^2}{\lambda_0} = \alpha (= z_{01}) \Rightarrow \quad W_{01}^2 = \frac{\lambda_0 \alpha}{\pi n}$$

To obtain the beam waist at mirror 2, we obtain the matrix from mirror 1 to mirror 2.

$$T = \begin{pmatrix} 1 & d_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix} \begin{pmatrix} 1 & d_1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 - \frac{d_2}{f} & d_1 + d_2(1 - d_1/f) \\ -\frac{1}{f} & 1 - \frac{d_1}{f} \end{pmatrix}$$

$$\frac{1}{q_2} = \frac{C + D\frac{1}{q_1}}{A + B\frac{1}{q_1}} \xrightarrow{q_1 = i z_{01}} \frac{1}{q_2} = \frac{i z_0 C + D}{i z_0 A + B} = \frac{BD + z_0^2 AC - i z_{01} \overbrace{(AD - BC)}^{1}}{B^2 + z_{01}^2 A^2}$$

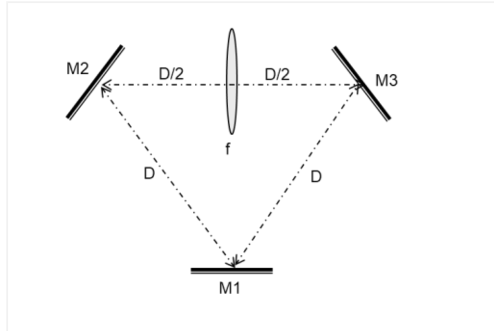
on the other hand, $\frac{1}{q_2} = \frac{1}{R_2} - i \frac{\lambda_0}{\pi W_2^2} \xrightarrow[\text{mirror}]{\text{flat}} 0 - i \frac{\lambda_0}{\pi W_{02}^2}$

$$\Rightarrow \frac{\lambda_0}{\pi W_{02}^2} = \frac{z_{01}}{B^2 + z_{01}^2 A^2} = \frac{z_{01}}{\left(d_1 + d_2(1 - d_1/f)\right)^2 + z_{01}^2 \left(1 - \frac{d_2}{f}\right)^2}$$

from here, W_{02} can be determined since z_{01} is known in terms of d_1, d_2 & f.

4.

Consider the ring laser cavity shown in the accompanying diagram.



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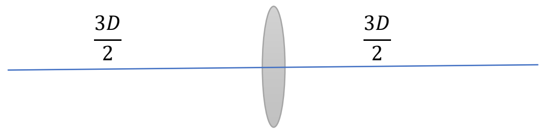

$$T = \begin{pmatrix} 1 & \frac{3D}{2} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix} \begin{pmatrix} 1 & \frac{3D}{2} \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 - \frac{3D}{2f} & \frac{3D}{2}\left(2 - \frac{3D}{2f}\right) \\ -\frac{1}{f} & 1 - \frac{3D}{2f} \end{pmatrix}$$

Stability:

$-1 < \frac{A+D}{2} < 1, \quad 0 < 1 - \frac{3D}{4f} < 1, \quad 0 < D < \frac{4f}{3}$

Beam waist is at the symmetry point (M1).

$Z_0 = \frac{B}{\sqrt{1 - \frac{(A+D)^2}{4}}} = \sqrt{3Df}\,(\sqrt{1 - 3D/4f})$ (Stability condition above also verified here)

$$w_0^2 = Z_0/\pi\lambda_0$$