Laser Physics I (PHYS/ECE 464), Fall 2022

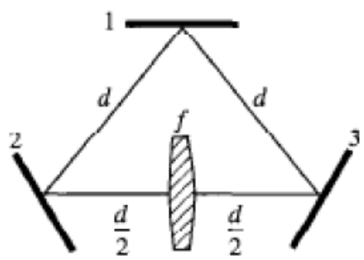
Homework #3, Due Monday, Sept. 19

1 Consider the ring laser cavity shown in the accompanying diagram.

(a) Show an equivalent-lens waveguide for this cavity and identify a unit cell starting just after the lens and proceeding counterclockwise around the triangle.

(b) What is the transmission matrix for this unit cell? (Demonstrate that you have the component matrices in proper order.)

(c) What are the values of d/f that make this a stable cavity?



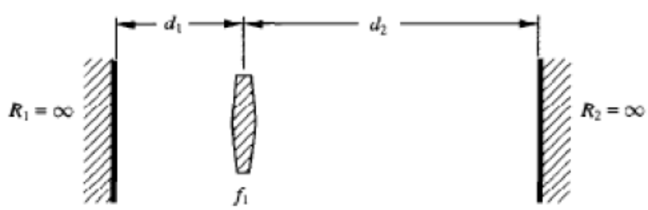
2 Consider the cavity shown in the accompanying diagram.

(a) Construct an equivalent-lens waveguide.

(b) Indicate a unit cell starting at a flat mirror, R_1.

(c) Find the ray matrix for the unit cell of (b).

(d) Discuss the stability of this cavity by constructing a diagram similar to Fig. 2.9.



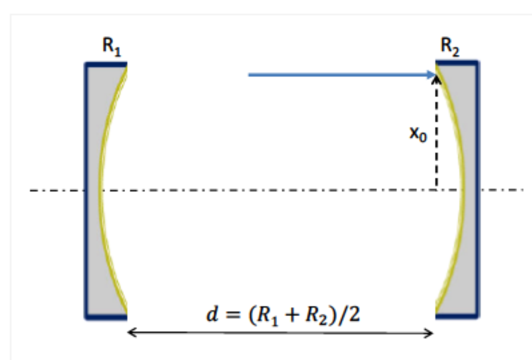
3. Consider the double concave <u>confocal cavity</u> shown below:

(a) Find the roundtrip ABCD matrix (Choose the starting point to be just before mirror 2).

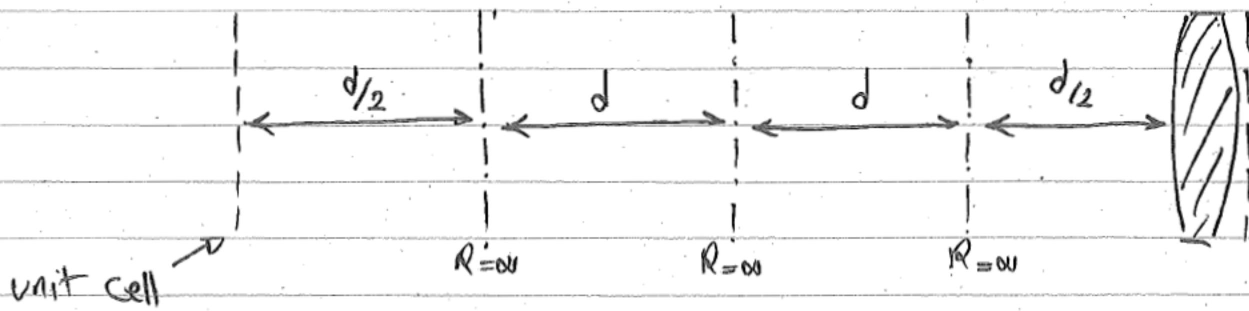
(b) Discuss the stability of this cavity for the symmetric $R_1=R_2$, and asymmetric ($R_1 \neq R_2$) cases.

(c) A ray parallel to optical axis is incident on mirror 2 at a distance x_0 -as shown. Derive an expression for the position $x(s)$ of this ray (on mirror 2) as a function of roundtrip number s. Discuss your results for cases $R_1=R_2$, $R_1>R_2$ and $R_1<R_2$.

(d) Draw the ray diagram for a few round trips in each case.



1 [redacted]) a →



b → $T = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}\begin{pmatrix} 1 & d/2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & d/2 \\ 0 & 1 \end{pmatrix}$

$$= \begin{pmatrix} 1 & 3d \\ -1/f & -d/f - 2d/f + 1 \end{pmatrix}$$

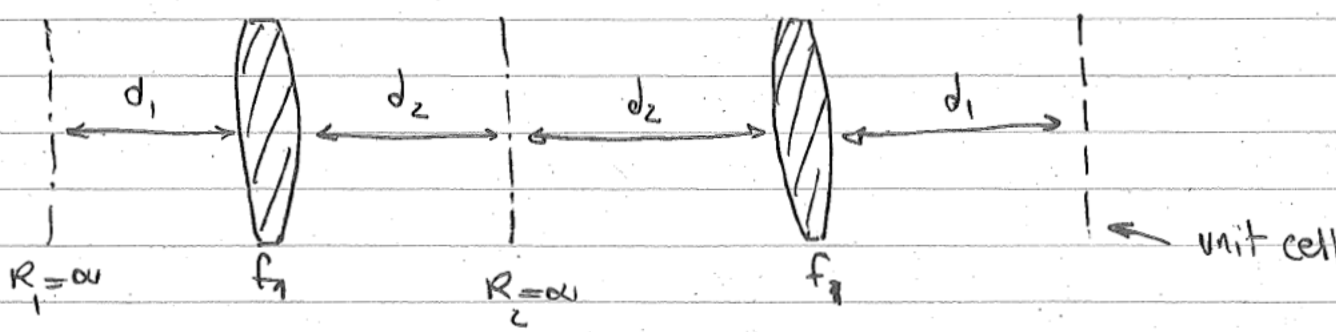
c → Laser stability Conds: $-1 < \frac{A+D}{2} < 1$

$\rightarrow -1 < \frac{1}{2}(1 - d/f - 2d/f + 1) < 1 \rightarrow -1 < -\frac{3}{2}\, d/f + 1 < 1$

$\rightarrow -2 < -\frac{3}{2}\, d/f < 0 \rightarrow 0 < d/f < \frac{4}{3}$

---

2 [redacted] a →



b → $T = \begin{pmatrix} 1 & d_1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -1/f_1 & 1 \end{pmatrix}\begin{pmatrix} 1 & d_2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & d_2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -1/f_1 & 1 \end{pmatrix}\begin{pmatrix} 1 & d_1 \\ 0 & 1 \end{pmatrix}$

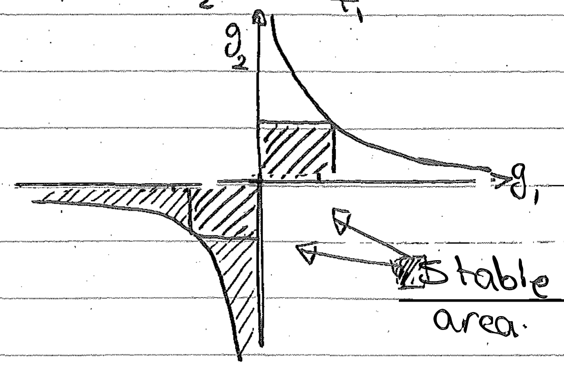
$$= \begin{pmatrix} 1 - \frac{2d_1}{f_1} - \frac{2d_2}{f_1} + \frac{2d_1 d_2}{f_1^2} & \left(1 - \frac{d_1}{f_1}\right)\left(1 - \frac{2d_2}{f_1} + 2d_1 d_2\right) - \frac{d_1}{f_1} + d_1^2 \\ -\frac{2}{f_1} + \frac{2d_2}{f_1^2} & 1 - \frac{2d_1}{f_1} - \frac{2d_2}{f_1} + \frac{2d_1 d_2}{f_1^2} \end{pmatrix}$$

$C \rightarrow -1 < \frac{A+D}{2} < 1 \Rightarrow -1 < 1 - \frac{2d_1}{f_1} - \frac{2d_2}{f_2} + \frac{2d_1 d_2}{f_1^2} < 1$

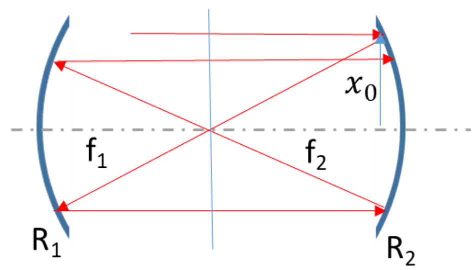
$0 < 1 - d_1/f_1 - d_2/f_1 + d_1 d_2 / f_1^2 < 1 \rightarrow$

$0 < d_1/f_1 + d_2/f_2 - d_1 d_2 / f_1^2 < 1 \rightarrow$

$0 < \underbrace{(1 - d_1/f_1)}_{g_1} \underbrace{(1 - d_2/f_1)}_{g_2} < 1$



#3 Solution:



(a)

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} -R/R_2 & (R_1^2 - R_2^2)/R_2R_1 \\ 0 & -R_2/R_1 \end{pmatrix}$$

(b)

$$\frac{A+D}{2} = -\frac{1}{2}\left(\frac{R_1}{R_2} + \frac{R_2}{R_1}\right)$$

$(A+D)/2$ is always negative and thus satisfies <1 condition, but it is also <-1 for $R_2 \neq R_1$. It is only marginally stable for $R_1 = R_2$ where $\frac{A+D}{2} = -1$.

(c) Starting from the ray position difference equation:

$$x^{s+2} - \frac{2(A+D)}{2}x^{s+1} + x^s = 0$$

Since the cavity is generally unstable, we seek a solution of the type $x(s) = x_0 Z^s$ (i.e. not sinusoidal). You will then find the solutions to the resulting quadratic equation

$$Z^2 - \frac{2(A+D)}{2}Z + 1 = 0$$

to be:

$$Z_{1,2} = -\frac{R_1}{R_2}\ , -\frac{R_2}{R_1}.$$

Pick the one that satifies the beam position (and or slope) in the first and second round trips (see also the procefure in the text), which gives:

$$x(s) = x_0\left(-\frac{R_1}{R_2}\right)^s$$

For $R_1 = R_2$, $x(s) = x_0$; that is the ray returns to its original position in every round trip (i.e. marginally stable).

For $R_1 > R_2$, ray position diverges and gets out of the cavity as s increase.
For $R_1 < R_2$, ray position converges towards the center (axis) exponentially. You may alternatively write

$$x(s) = x_0(-1)^s e^{-Ks} \text{ , where } \quad \mathrm{K} = \ln\left(\frac{R_2}{R_1}\right).$$