Laser Physics I (PHYS/ECE 464)
Homework #1, Due Wed., Sept. 7
Fall 2022

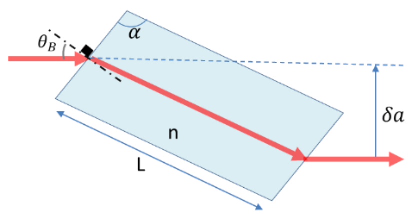
From *Verdeyen*:

1. Problem 1.4

2. Problem 1.5

3. Problem 1.7

4. A laser crystal (having refractive index n) and length L is Brewster cut such that the incident beam at θ_B emeges parallel to the crystal sides as shown in the Fig.



What is the angle α? What is the the beam's lateral deviation (δa) in terms of n and L?

5. Show that:

Photon energy $E(eV)\sim 1.24/\lambda(\mu m)$,

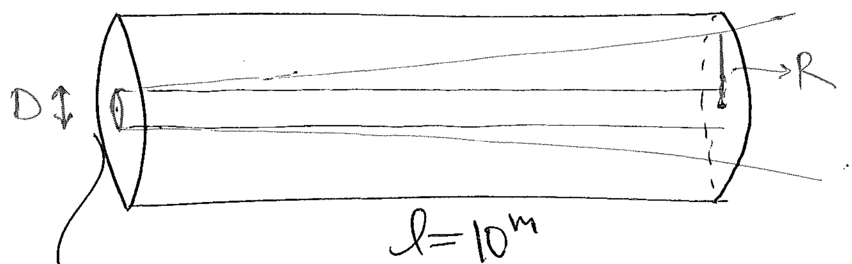
Electric field $E_0(V/cm)\sim 27[I(W/cm^2)/n]^{1/2}$,

where λ is the wavelength, I is the irradiance and n is the refractive index. *Try to memorize these useful relations.*

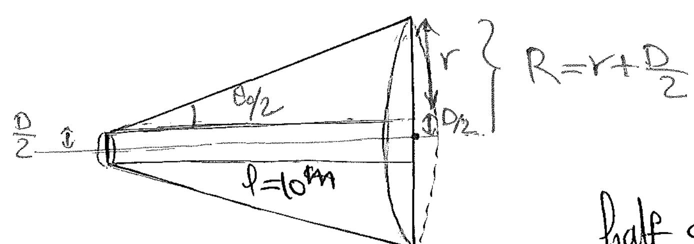
Solutions of Homework # 1

1. (8 Points)

Problem # 1.4 , Verdeyen 3^{rd} ed.



beam of diameter D enters the tube from here.



half of the angle of divergance

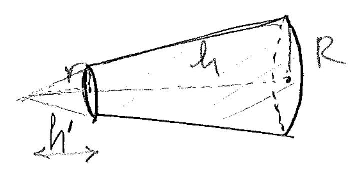
According to Eq. (1.7.1), $\frac{\theta_0}{2} = \frac{\lambda}{\pi D/2} = \frac{2\lambda}{\pi D}$

Since angles are small, $\rightarrow \tan\frac{\theta_0}{2} \cong \frac{\theta_0}{2} = \frac{r}{l} = \frac{2\lambda}{\pi D} \Rightarrow \boxed{r = \frac{2\lambda l}{\pi D}}$

Thus, the radius of the base of the cone : $R = r + \frac{D}{2} = \frac{2\lambda l}{\pi D} + \frac{D}{2}$

Volume of an incomplete cone:

$$V = \frac{1}{3}\pi h (R^2 + r^2 + rR)$$



Now, find the minimum, $\frac{dV}{dD} = \frac{1}{3}\pi l\left(\frac{3}{4}2D + \left(\frac{2l\lambda}{\pi}\right)^2 \cdot \frac{-2}{D^3}\right) = 0$

$\left(\text{Also } \frac{d^2V}{dD^2} > 0\right)$

$$\Rightarrow \frac{3}{2}D = \frac{2}{D^3}\left(\frac{2l\lambda}{\pi}\right)^2$$

$$\Rightarrow D^4 = \frac{4}{3}\left(\frac{2l\lambda}{\pi}\right)^2$$

Substitute $l = 10^m$, $\lambda = 632.8^{nm}$ $\rightarrow$ $D = 0.0022^{m} = 0.22^{cm}$

2 (4 Points)

Problem #(1.5) Verdeyen 3rd ed.

$$E^{(J)} = h\nu = \frac{hc}{\lambda(m)} \Rightarrow E^{(eV)} = \frac{h\,c}{e}\frac{1}{\lambda(m)} = \frac{1.24\times10^{-6}}{\lambda(10^{-6}\mu m)} = \frac{1.24}{\lambda(\mu m)}$$

$h \to 6.63\times10^{-34}$ (J·s), $c \to 3\times10^{8}$ (m/s), $e \to 1.6\times10^{-19}$ (C)

Thus, $$\boxed{E^{(eV)} = \frac{1.24}{\lambda(\mu m)}}$$

$$E^{(J)} = h\nu(Hz) \Rightarrow E^{(eV)} = \frac{h}{e}\nu(Hz) \Rightarrow \boxed{E^{(eV)} = 4.143\times10^{-15}\,\nu(Hz)}$$

$$E^{(eV)} = \frac{1.24\times10^{-6}}{\lambda(m)} = 1.24\times10^{-4}\frac{1}{\lambda(cm)} \to = \bar{\nu}(cm)$$ spectroscopic definition of wavenumber.

$$\Rightarrow \boxed{E^{(eV)} = 1.24\times10^{-4}\,\bar{\nu}(cm)}$$

Using the boxed equations it is easy to convert the quantities.

Source	eV	λ(A°)	λ(nm)	ν (Hz)	$\bar{\nu}$ (cm^{-1})
GaAs	1.47	8435	843.5	3.55×10^{14}	11,855
Ar^+	2.41	5145	514.5	5.81×10^{14}	19,435
He-Ne	1.96	6328	632.8	4.73×10^{14}	15,806
CO_2	0.117	105983	10598.3	2.83×10^{13}	943
ISM band	5.61×10^{-8}	22.1 M	22.1 m	13.56 MHz	4.52×10^{-4}
KrF	5	2490	249	1.2×10^{15}	40,323

3 (3 Points)

Problem #(1.7), Verdeyen 3rd ed.

$$E(k_y) = \int_{-\infty}^{+\infty} E(y)\, e^{-ik_y y}\, dy$$

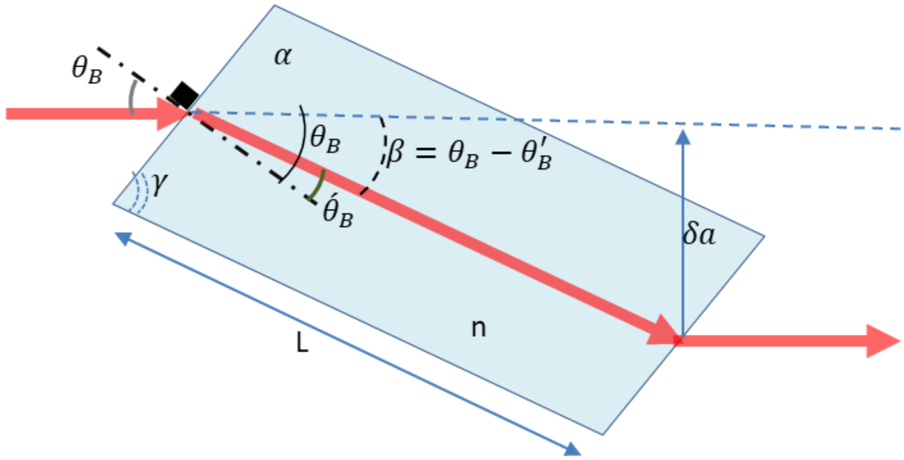
$$= \int_{-\infty}^{+\infty} E_0\, e^{-\left(\frac{y}{w_0}\right)^2} e^{-ik_y y}\, dy$$

The exponent is, $-\left(\frac{y}{w_0}\right)^2 - ik_y y = -\left(\frac{y}{w_0} + \frac{iw_0 k_y}{2}\right)^2 - \frac{w_0^2 k_y^2}{2^2}$

$$\text{Thus } E(k_y) = E_0\, e^{-\left(\frac{w_0 k_y}{2}\right)^2} \underbrace{\int_{-\infty}^{+\infty} e^{-\left(\frac{y}{w_0} + \frac{iw_0 k_y}{2}\right)^2} dy}_{\pi w_0 \ \left(\text{use } \int_0^{\infty} e^{-ax^2} dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}\right)}$$

$$\Longrightarrow E(k_y) = \sqrt{\pi}\, E_0 w_0\, e^{-\left(\frac{w_0 k_y}{2}\right)^2}$$

4.



(a)

$\alpha = \pi - \gamma = \pi - (\pi/2 - \theta_B') = \pi - \theta_B$; note $\gamma = \theta_B$

$$\alpha = \pi - \tan^{-1} n$$

(b)

$$\delta a = L \sin\beta = L\sin\left(\theta_B - \theta_B'\right) = L\sin(2\theta_B - \pi/2)$$
$$= -L\cos(2\theta_B) = -L(2\cos{\theta_B}^2 - 1) = -L\left(\frac{2}{1+\tan{\theta_B}^2} - 1\right)$$
$$= L\,\frac{n^2-1}{n^2+1}$$

(2 Points)

(5) Show that.

$E(eV) \simeq \frac{1.24}{\lambda(\mu m)}$ → already shown in problem 2.

Electric field $E_0\left(\frac{V}{cm}\right) \simeq 27\left(I\left(\frac{W}{cm^2}\right)/n\right)^{1/2}$

$I = \langle S \rangle = \frac{1}{2\mu_0 c} n |E_0|^2 = \frac{1}{2} c\epsilon_0 n |E_0|^2$

$\Rightarrow E_0\left(\frac{V}{m}\right) = \left[\frac{2}{c\epsilon_0 n} I\left(\frac{W}{m^2}\right)\right]^{1/2} = 27.44\left(\frac{I\left(\frac{W}{m^2}\right)}{n}\right)^{1/2}$

$c \to 3\times10^8 \frac{m}{s}$, $\epsilon_0 \to 8.85\times10^{-12} \frac{F}{m}$

$\Rightarrow E_0\left(\frac{V}{10^{-2}cm}\right) = 27.44\left(\frac{I\left(\frac{W}{10^{-4}cm^2}\right)}{n}\right)^{1/2}$

$\rightsquigarrow E_0\left(\frac{V}{cm}\right) \simeq 27\left(\frac{I\left(\frac{W}{cm^2}\right)}{n}\right)^{1/2}$