Final Formula Sheet
PHYS/ECE 464 (Laser Physics I)- University of New Mexico (2021)

Hermite-Gaussian Beams

$$\frac{E(x,y,z)}{E_0} = H_m\left(\frac{\sqrt{2}x}{w(z)}\right)H_p\left(\frac{\sqrt{2}y}{w(z)}\right)\frac{w_0}{w(z)}exp\left(-i\frac{kr^2}{2q(z)}\right)\times exp(-i[kz-(1+m+p)\,tan^{-1}(z/z0)])$$

$$\frac{1}{q(z)} = \frac{1}{R(z)} - i\frac{\lambda_0}{\pi n w^2(z)};\ q(z) = z + iz_0\ ;\ w^2(z) = w_0^2\left(1+\frac{z^2}{z_0^2}\right);\ \ R(z) = z\left(1+\frac{z_0^2}{z^2}\right);\ \ z_0 = \frac{\pi n w_0^2}{\lambda_0},\ k = n\frac{\omega}{c} = \frac{2\pi n}{\lambda_0}$$

Irradiance: $I = \langle S \rangle = \frac{nc\varepsilon_0}{2}{E_0}^2$

Fresnel's reflectivities:

$$r_{||} = \frac{n_t\cos(\theta_i) - n_i\cos(\theta_t)}{n_t\cos(\theta_i) + n_i\cos(\theta_t)} = \frac{tan(\theta_i-\theta_t)}{tan(\theta_i+\theta_t)}$$

$$r_{\perp} = -\frac{n_i\cos(\theta_i) - n_t\cos(\theta_t)}{n_i\cos(\theta_i) + n_t\cos(\theta_t)} = -\frac{sin(\theta_i-\theta_t)}{sin(\theta_i+\theta_t)}$$

Brewster angle (from 1 to 2): $\theta_B = tan^{-1}(n_2/n_1)$

Critical angle (from 1 to 2): $\theta_c = sin^{-1}(n_1/n_2)$

$n \rightarrow \tilde{n} = n + i\kappa$ *when complex*

Snell's Law $n_i\,sin(\theta_i) = n_t\,sin(\theta_t)$

Intensity (Power) reflectivity: $R=|r|^2$

$R = (n_t-n_i)^2/(n_t+n_i)^2$ when $\theta_i = 0$

Lens Transformation of a Gaussian beam: $\frac{1}{R_{out}} = \frac{1}{R_{in}} - \frac{1}{f}$	Lens-makers' formula: $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Fabry-Perot Transmission and Reflection (for gain G_0 >1, loss A_0<1, passive G_0=A_0=1)

$$T(\theta, G_0) = \frac{G_0(1-R_1)(1-R_2)}{\left(1-G_0\sqrt{R_1R_2}\right)^2 + 4G_0\sqrt{R_1R_2}\,sin^2(\theta)}$$

$$R(\theta, G_0) = \frac{\left(\sqrt{R_1}-G_0\sqrt{R_2}\right)^2 + 4G_0\sqrt{R_1R_2}\,sin^2(\theta)}{\left(1-G_0\sqrt{R_1R_2}\right)^2 + 4G_0\sqrt{R_1R_2}\,sin^2(\theta)}$$

Finesse $F = \frac{\pi\sqrt[4]{R_1R_2}}{1-\sqrt{R_1R_2}} = \frac{\Delta\nu_{FSR}}{\Delta\nu_{1/2}} = \frac{\Delta\lambda_{FSR}}{\Delta\lambda_{1/2}}$

Free Spectral Range: $\Delta\nu_{FSR} = \frac{c}{2nd} = \frac{1}{\tau_{RT}}$

$$\theta = kd = \frac{2\pi\nu nd}{c}$$

Passive Symmetric Fabry-Perot:

$$T(\theta) = \frac{1}{1+\left(\frac{2}{\pi}F\,sin(\theta)\right)^2}\ ,\quad F = \frac{\pi\sqrt{R}}{1-R}$$

$$\frac{\Delta\lambda}{\lambda} = \frac{\Delta\nu}{\nu},\ \nu = c/\lambda$$

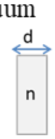
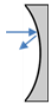
Photon Lifetime:

$$\tau_p = \frac{\tau_{RT}}{1-R_1R_{21}} \approx \frac{1}{2\pi\Delta\nu_{1/2}}$$

Resonance Condition:

Roundtrip phase change= $q2\pi$, or $\theta = q\pi$ (q=integer)

ABCD Matrices $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$ AD-BC=1 (or $=n_1/n_2$) $\begin{pmatrix} r_2 \\ r_2' \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} r_1 \\ r_1' \end{pmatrix}$

Free space of length d $\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$	Dielectric interface (from n_1 to n_2) $\begin{pmatrix} 1 & 0 \\ 0 & n_1/n_2 \end{pmatrix}$	ABCD rule for Gaussian Beams $q_2 = \frac{Aq_1+B}{Cq_1+D}$ where	$q(z) = z + iz_0$ or $\frac{1}{q(z)} = \frac{1}{R(z)} - i\frac{\lambda_0}{\pi n w(z)}$
medium of length d and index n_2=n immersed in vacuum (n_1=1). $\begin{pmatrix} 1 & d/n \\ 0 & 1 \end{pmatrix}$  d n	Thin lens of focal length f $\begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$  $f>0$	Cavity Stability Condition: $-1 < \frac{A+D}{2} < 1$ q at the starting point of the cavity with a roundtrip ABCD: $\frac{1}{q} = -\frac{A-D}{2B} - i\frac{\sqrt{1-(A+D)^2/4}}{B}$	
Mirror with radius of curvature R $\begin{pmatrix} 1 & 0 \\ -2/R & 1 \end{pmatrix}$  $R>0$	Spherical dielectric interface $\begin{pmatrix} 1 & 0 \\ (1-n_1/n_2)/R & n_1/n_2 \end{pmatrix}$	Photon Density (Photon Number per Volume) $\frac{N_p}{V} = \frac{I}{h\nu c/n_g}$	

Einstein's coefficients: $\frac{A_{21}}{B_{21}} = \frac{8\pi n^3 h\nu^3}{c^3}$, $g_2 B_{21} = g_1 B_{12}$

In thermal equilibrium (no pumping): $\frac{N_2}{N_1} = \frac{g_2}{g_1} e^{-(E_2-E_1)/k_B T}$

Gain in 2-level system: $\gamma(\nu) = \sigma(\nu)\left[N_2 - \frac{g_2}{g_2} N_1\right]$;

Gain cross-section: $\sigma(\nu) = A_{21} \frac{\lambda^2}{8\pi n^2} g(\nu)$

Lineshape Normalization $\int g(\nu)\, d\nu = 1$;

Beer's Law: $\frac{dI}{dz} = +\gamma I = -\alpha I$

Lorentzian lineshape: $g(\nu) = \frac{\Delta\nu_h/2\pi}{(\nu-\nu_0)^2 + (\Delta\nu_h/2)^2}$

Doppler broadened line shape

$g(\nu) = \left(\frac{4\,ln\,2}{\pi}\right)^{1/2} \frac{1}{\Delta\nu_D} exp\left[(-4\,ln\,2)\left(\frac{\nu-\nu_0}{\Delta\nu_D}\right)^2\right]$ with $\Delta\nu_D = \left(\frac{8kT\,ln\,2}{Mc^2}\right)^{1/2} \nu_0$

$\frac{dN_p}{dt} = \frac{G^2 S - 1}{\tau_{RT}} N_p + N_2 c\sigma$ (Photon number dynamics due to *stimulated* and *spontaneous* emission)

S (roundtrip survival factor) $=R_1R_2\ldots$, G^2=roundtrip gain $=e^{2g}$, with $g=\gamma L_g$ (integrated gain).
Threshold condition: $SG^2=1$ (linear cavity), $SG=1$ (ring cavity)

Schawlow-Townes limit for laser linewidth: $\Delta\nu_{osc} \approx 2\pi \frac{h\nu}{P_{out}} (\Delta\nu_{1/2})^2$

At steady-state: $\boldsymbol{\gamma = \gamma_{th} = \frac{\gamma_0}{1+I/I_s}}$ (for homogenously broadened)
Inside the gain medium: $I \approx I^+ + I^- \approx 2I^+$ for a high-Q linear (standing-wave) or bidirectional ring cavity, $I \approx I^+$ for a unidirectional ring cavity.

$\mathbf{I_{out}=T_a..T_2 I^+}$ (T_2 is the output coupling transmission and $T_a\ldots$ are the transmission of other optical surfaces in the path).

Optimum output coupling: $\mathbf{T_2^{opt} = -L_i + (g_0 L_i)^{1/2}}$ where L_i accounts for roundtrip internal (useless) losses , $g_0=\gamma_0 l_g$ is the unsaturated (small signal) integrated gain. l_g is the length of the gain medium.

Q-Switching and Gain-Switching: $\Delta t_p \approx \tau_p$ (cavity photon lifetime)

Modelocking: Repetition Rate $=1/\tau_{RT} = c/2n_g L$ (linear cavity), n_g (group index), Pulsewidth: $\Delta t_p \approx > 1/\Delta\nu$

Threshold current density in a diode laser: $J_{th} = eN_{eh}^{th} d/\tau_r$

Physical Constants

$c \sim 3 \times 10^8$ m·s^{-1}	$h=6.63 \times 10^{-34}$ J·s	$e=1.6 \times 10^{-19}$ C
$k_B=1.380 \times 10^{-23}$ J·K^{-1}	$\varepsilon_0=8.85\times10^{-12}$ F/m	$m_e=9.1 \times 10^{-31}$ kg

1G = 10^{-4} T , 1 eV = 1.602×10^{-19} J, 1 dyne = 10^{-5} N, 1 erg = 10^{-7} J