Final Exam Formula Sheet
PHYS/ECE 464 (Laser Physics I)- University of New Mexico (2022)

Hermite-Gaussian Beams

$$\frac{E(x,y,z)}{E_0} = H_m\left(\frac{\sqrt{2}x}{w(z)}\right) H_p\left(\frac{\sqrt{2}y}{w(z)}\right)\frac{w_0}{w(z)} exp\left(-i\frac{kr^2}{2q(z)}\right) \times exp(-i[kz-(1+m+p)\,tan^{-1}(z/z0)])$$

$$\frac{1}{q(z)} = \frac{1}{R(z)} - i\frac{\lambda_0}{\pi n w^2(z)}, \quad w^2(z) = w_0^2\left(1+\frac{z^2}{z_0^2}\right), \quad R(z) = z\left(1+\frac{z_0^2}{z^2}\right), \quad z_0 = \frac{\pi n w_0^2}{\lambda_0} \quad k = n\frac{\omega}{c} = \frac{2\pi n}{\lambda_0}$$

Irradiance: $I = \langle S \rangle = \frac{nc\varepsilon_0}{2}{E_0}^2$

Snell's Law $n_i \sin(\theta_i) = n_t \sin(\theta_t)$

Fresnel's Reflectivities:

$$r_{||} = \frac{n_t\cos(\theta_i) - n_i\cos(\theta_t)}{n_t\cos(\theta_i) + n_i\cos(\theta_t)} = \frac{tan(\theta_i-\theta_t)}{tan(\theta_i+\theta_t)}$$

$$r_{\perp} = -\frac{n_i\cos(\theta_i) - n_t\cos(\theta_t)}{n_i\cos(\theta_i) + n_t\cos(\theta_t)} = -\frac{sin(\theta_i-\theta_t)}{sin(\theta_i+\theta_t)}$$

Intensity (Power) reflectivity: $R=|r|^2$

Brewster angle (from 1 to 2): $\theta_B = tan^{-1}(n_2/n_1)$

Critical angle (from 1 to 2): $\theta_c = sin^{-1}(n_1/n_2)$

$n \to \tilde{n} = n + i\kappa$

Lens Transformation/Gaussian beam: $\frac{1}{R_{out}} = \frac{1}{R_{in}} - \frac{1}{f}$	$\frac{1}{f} = (n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ Lens-makers' formula:

Fabry-Perot Transmission and Reflection (for gain G_0 >1, loss A_0<1, passive $G_0=A_0=1$)

$$T(\theta, G_0) = \frac{G_0(1-R_1)(1-R_2)}{\left(1-G_0\sqrt{R_1R_2}\right)^2 + 4G_0\sqrt{R_1R_2}\,sin^2(\theta)}$$

$$R(\theta, G_0) = \frac{\left(\sqrt{R_1}-G_0\sqrt{R_2}\right)^2 + 4G_0\sqrt{R_1R_2}\,sin^2(\theta)}{\left(1-G_0\sqrt{R_1R_2}\right)^2 + 4G_0\sqrt{R_1R_2}\,sin^2(\theta)}$$

Finesse $F = \frac{\pi\sqrt[4]{R_1R_2}}{1-\sqrt{R_1R_2}} = \frac{\Delta\nu_{FSR}}{\Delta\nu_{1/2}} = \frac{\Delta\lambda_{FSR}}{\Delta\lambda_{1/2}}$

Free Spectral Range: $\Delta\nu_{FSR} = \frac{c}{2nd} = \frac{1}{\tau_{RT}}$

$\theta = kd = \frac{2\pi\nu nd}{c}\cos(\phi)$, $\phi = internal\ angle\ in\ FP$

Passive Symmetric Fabry-Perot:

$T(\theta) = \frac{1}{1+\left(\frac{2}{\pi}F\,sin(\theta)\right)^2}$, $F = \frac{\pi\sqrt{R}}{1-R}$ *(Finesse)*

Photon Lifetime:

$$\tau_p = \frac{\tau_{RT}}{1-S} \approx \frac{1}{2\pi\Delta\nu_{1/2}}$$

$S = R_1R_2$.. *(survival factor)*

Resonance Condition: $\theta = q\pi$ *(q=integer)*

$$\frac{\Delta\nu}{\nu_0} = \frac{\Delta\lambda}{\lambda_0} = \frac{\Delta d}{d} = \frac{\Delta n}{n} = \frac{\Delta(\cos(\phi)}{(\cos(\phi)}$$

ABCD Matrices $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$ AD-BC=1 $\begin{pmatrix} r_2 \\ r_2' \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} r_1 \\ r_1' \end{pmatrix}$

Free space of length d $\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$

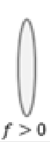
Dielectric interface (from n_1 to n_2) $\begin{pmatrix} 1 & 0 \\ 0 & n_1/n_2 \end{pmatrix}$

ABCD rule for Gaussian Beams

$q_2 = \frac{Aq_1+B}{Cq_1+D}$ *where* $q(z) = z + iz_0$ *or* $\frac{1}{q(z)} = \frac{1}{R(z)} - i\frac{\lambda_0}{\pi n w^2(z)}$

medium of length d and index n_2=n immersed in vacuum (n_1=1). $\begin{pmatrix} 1 & d/n \\ 0 & 1 \end{pmatrix}$

Thin lens of focal length f $\begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$



Cavity Stability Condition: $-1 < \frac{A+D}{2} < 1$

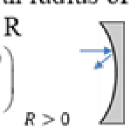
q at the starting point of the cavity with a roundtrip ABCD:

$$\frac{1}{q} = -\frac{A-D}{2B} - i\frac{\sqrt{1-(A+D)^2/4}}{B}$$

Stability condition for 2-mirror cavity with R_1, R_2, and length d

$0 < g_1\,g_2 < 1$ where $g_{1,2} = 1 - d/R_{1,2}$

Mirror with radius of curvature R $\begin{pmatrix} 1 & 0 \\ -2/R & 1 \end{pmatrix}$



Spherical dielectric interface $\begin{pmatrix} 1 & 0 \\ (1-n_1/n_2)/R & n_1/n_2 \end{pmatrix}$

Photon Density (Photon Number per Volume)

$$\frac{N_p}{V} = \frac{I}{h\nu c/n_g}$$

Einstein's relations: $\frac{A_{21}}{B_{21}} = \frac{8\pi n^3 h\nu^3}{c^3}$; $g_1 B_{12} = g_2 B_{21}$

Gain and Absorption: $\gamma(\nu) = \sigma_e(\nu)\left(N_2 - \frac{g_2}{g_1} N_1\right)$, $\alpha(\nu) = \sigma_a(\nu)\left(N_1 - \frac{g_1}{g_2} N_2\right)$; $g_{1,2} = degeneracies$

Emission (gain) and absorption cross-sections: $\sigma_a(\nu) = \frac{g_2}{g_1}\sigma_e(\nu)$; $\sigma_e(\nu) = A_{21}\frac{\lambda^2}{8\pi n^2} g(\nu)$

Line shape: $\int g(\nu) d\nu = 1$; $g(\nu_0) \approx 1/\Delta\nu$

Beer's Law: $\frac{1}{I}\frac{dI}{dz} = +\gamma I$ or $\frac{1}{I}\frac{dI}{dz} = -\alpha I$

Gain (or absorption) saturation in homogenously broadened media: $\gamma = \frac{\gamma_0}{1+I/I_s}$, with $I_s(\nu) = h\nu/\sigma(\nu)\tau_2$

Amplifier Gain: $\ln\left(\frac{I_2}{I_1}\right) + \frac{I_2 - I_1}{I_s} = \gamma_0 l_g$

Lorentzian line shape: $g(\nu) = \frac{\Delta\nu_h/2\pi}{(\nu-\nu_0)^2 + (\Delta\nu_h/2)^2}$

Doppler broadened line shape:

$$g(\nu) = \left(\frac{4\ln 2}{\pi}\right)^{1/2} \frac{1}{\Delta\nu_D} exp\left[(-4\ln 2)\left(\frac{\nu-\nu_0}{\Delta\nu_D}\right)^2\right] \text{ with } \Delta\nu_D = \left(\frac{8kT\ln 2}{Mc^2}\right)^{1/2} \nu_0$$

$\frac{dN_p}{dt} = \frac{G^2 S - 1}{\tau_{RT}} N_p + N_2 c\sigma$ (Photon number dynamics due to *stimulated* and *spontaneous* emission)

Roundtrip survival factor: $S = R_1 R_2 \ldots$, Total cavity losses $L \approx 1 - S \approx T_2 + L_i$

G^2=roundtrip gain $=e^{2g}$, with $g = \gamma l_g$ (integrated gain).

Threshold and steady state condition: $SG^2=1$ (linear cavity), $SG=1$ (ring cavity),

Schawlow-Townes limit for laser linewidth: $\Delta\nu_{osc} \approx 2\pi \frac{h\nu}{P_{out}} (\Delta\nu_{1/2})^2$

At steady-state: $\gamma = \gamma_{th} = \frac{\gamma_0}{1+I/I_s}$ (for homogenously broadened), I= total intensity inside the gain

$I \approx I^+ + I^- \approx 2I^+$ for a high-Q standing-wave or bidirectional ring cavity,

$I \approx I^+$ for a unidirectional ring cavity.

$I_{out} = T_a..T_2 I^+$ (T_2 is the output coupling transmission and $T_a\ldots$ are the transmission of other optical surfaces in the path).

Optimum output coupling: $T_2^{opt} = -L_i + (g_0 L_i)^{1/2}$ where L_i accounts for roundtrip internal (useless) losses , $g_0 = \gamma_0 l_g$ is the unsaturated (small signal) integrated gain. l_g is the length of the gain medium.

Q-Switching and Gain-Switching: $\Delta t_p \approx \tau_p$ (cavity photon lifetime)

Modelocking: Repetition Rate $=1/\tau_{RT} = c/2n_g L$ (linear cavity), n_g (group index), Pulsewidth: $\Delta t_p \approx> 1/\Delta\nu$

Threshold current density in a diode laser: $J_{th} = eN_{eh}^{th} d/\tau_r$

Physical Constants

$c \sim 3 \times 10^8$ m·s^{-1}	$h = 6.63 \times 10^{-34}$ J·s	$e = 1.6 \times 10^{-19}$ C
$k_B = 1.380 \times 10^{-23}$ J·K^{-1}	$\varepsilon_0 = 8.85 \times 10^{-12}$ F/m	$m_e = 9.1 \times 10^{-31}$ kg

1 eV = 1.602×10^{-19} J, $hc/k_B \approx 1.44$ cmK, $\lambda(\mu m) \approx 1.24/E(eV)$