Laser Physics I (PHYC/ECE 464)
FALL 2014

OPTICAL SCIENCE & ENGINEERING
University of New Mexico

Midterm Exam, Closed Book, Closed Notes

Time: 5:30 – 7:00 pm

NAME ....
last *first*

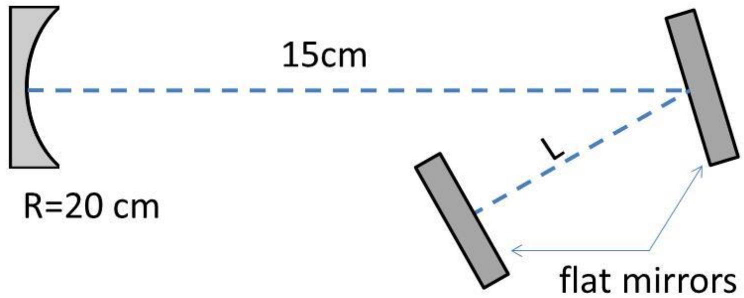
Score

Total= 100 points

Please staple and return these pages with your exam.

Instructor: M. Sheik-Bahae

1. **Cavity Mode:** In the 3-mirror cavity shown, the length L between the two flat mirrors is adjustable.

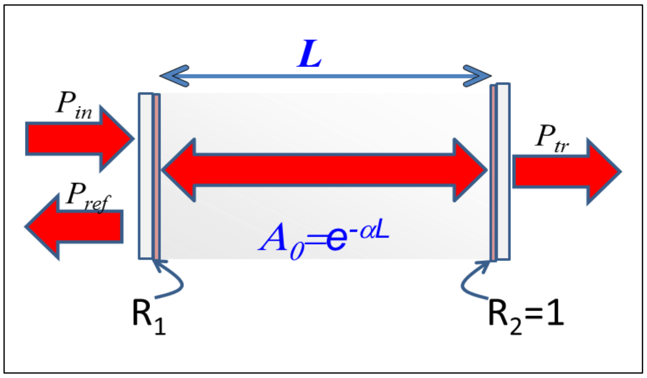


(a) Find the location the size of the minimum beam waist (w_0) as a function of L. (Hint: It is easier to employ the method of matching the phase front curvature rather than using ABCD) 15 points

(b) What is the range of L for which the cavity is stable? 5 points

Fabry-Perot with absorption

Consider a Fabry-Perot whose back mirror has a reflectivity of 100% (R_2=1) and R_1<1. (Such a device is known as Gires–Tournois Interferometer or GTI). The inside of this interferometer is filled with a gas with an absorption coefficient α as shown in the figure.

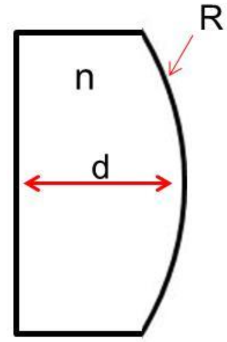


(a) Plot reflection (R) and transmission (T) as a function of θ=kL for no loss condition (α=0) for a light beam incident from left as shown. (Does the value of R_1 matter for these plots?) (5points)

(b) Fix R_1=0.9 and plot R and T (as in part a) for αL=0.1 and 0.2. Explain the features. (10 points)

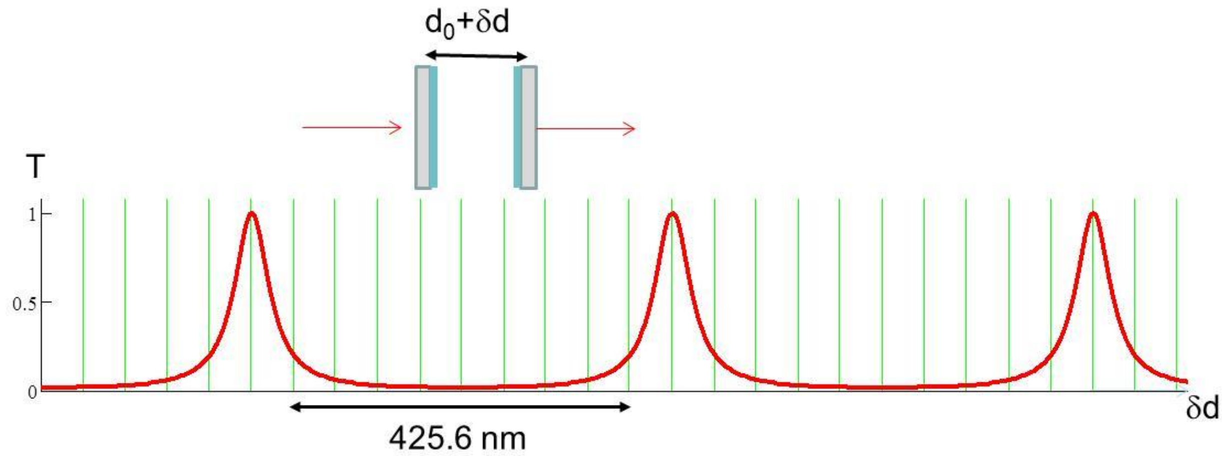
(c) Now fix αL=0.1 and tune FP to resonance by keeping L=mλ/2 (m=integer). Write the expression for R and T as a function of the front reflectivity R_1. Plot R and T vs. R_1 varying between 0 to 1. Something interesting should happen somewhere in this range. Identify it and explain your results in terms of absorption of light in the filling material.

(a) Derive the ray matrix for the “thick” plano-convex lens shown below: *(15 points)*



3. ***(20 points)***

Drawn to scale in the graph below is the power transmission of a scanning Fabri-Perot as the distance is increased from its intial 1cm to 1cm+1.44μm. The source is a single wavelength laser at wavelength λ_0.



a) What is λ_0? *(5pts.)*

b) What is $\Delta\nu_{1/2}$ (in MHz)? *(5pts.)*

c) What is the finesse? *(5pts.)*

d) What is the photon lifetime? *(5pts.)*

4. ***(25 points)*** Consider a pressure-broadened gaseous two-level medium with the following property:

- *Spontaneous emission lifetime:* $\tau_{sp}=1\ \mu s$
- *Homogeneous linewidth* $\Delta\nu_h=1.5$ *THz*
- *Line center wavelength:* $\lambda_0 = 5\ \mu m$
- *Molecular density (concentration):* $N_{total} = 2.5\times10^{19}\ cm^{-3}$
- *Non-degeneracy factors:* $g_1=5$, $g_2=1$

(a) What is the absorption coefficient $\alpha(\text{cm}^{-1})$ at the line center (5 μm) when all the molecules are in their ground state (level 1)? *(12.5 pts.)*

(b) What fraction of the molecules needs to be excited into level 2 in order to make this gas transparent (i.e. the onset of gain) at 5 μm? *(12.5pts.)*

Midterm Formula Sheet (Fall 2012)
PHYC/ECE 464 (Laser Physics I)- University of New Mexico
Instructor: Mansoor Sheik-Bahae

Hermite-Gaussian Beams:

$$\frac{E(x,y,z)}{E_0} = H_m\left(\frac{\sqrt{2}x}{w(z)}\right) H_p\left(\frac{\sqrt{2}y}{w(z)}\right)\frac{w_0}{w(z)}\exp\left(-i\frac{kr^2}{2q(z)}\right)\times\exp\left(-i\left[kz-(1+m+p)\tan^{-1}(z/z0)\right]\right)$$

$$\frac{1}{q(z)} = \frac{1}{R(z)} - i\frac{\lambda}{\pi w^2(z)}, \qquad w^2(z) = w_0^2\left(1+\frac{z^2}{z_0^2}\right), \qquad R(z) = z\left(1+\frac{z_0^2}{z^2}\right), \qquad z_0 = \frac{\pi n w_0^2}{\lambda_0}$$

$$k = n\frac{\omega}{c} = \frac{2\pi n}{\lambda_0}$$

Irradiance: $I = \langle S \rangle = \frac{nc\varepsilon_0}{2}E_0^2$

Snell's Law: $n_i \sin\theta_i = n_t \sin\theta_t$

Fresnel:

$$r_\parallel = \frac{n_t\cos\theta_i - n_i\cos\theta_t}{n_t\cos\theta_i + n_i\cos\theta_t} = \frac{\tan(\theta_i-\theta_t)}{\tan(\theta_i+\theta_t)} \qquad r_\perp = -\frac{n_i\cos\theta_i - n_t\cos\theta_t}{n_i\cos\theta_i + n_t\cos\theta_t} = -\frac{\sin(\theta_i-\theta_t)}{\sin(\theta_i+\theta_t)}$$

$$t_\parallel = \frac{2n_i\cos\theta_i}{n_i\cos\theta_t + n_t\cos\theta_i} = \frac{2\sin\theta_t\cos\theta_i}{\sin(\theta_i+\theta_t)\cos(\theta_i-\theta_t)} \qquad t_\perp = \frac{2n_i\cos\theta_i}{n_i\cos\theta_i + n_t\cos\theta_t} = \frac{2\sin\theta_t\cos\theta_i}{\sin(\theta_i+\theta_t)}$$

when there is total internal reflection at an air or vacuum interface:

$$r_\parallel = \frac{\cos\theta_i - in_i\sqrt{n_i^2\sin^2\theta_i - 1}}{\cos\theta_i + in_i\sqrt{n_i^2\sin^2\theta_i - 1}} \qquad r_\perp = \frac{n_i\cos\theta_i - i\sqrt{n_i^2\sin^2\theta_i - 1}}{n_i\cos\theta_i + i\sqrt{n_i^2\sin^2\theta_i - 1}}$$

Lens-maker's formula:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Lens Transformation of a Gaussian beam:

$$\frac{1}{R_{out}} = \frac{1}{R_{in}} - \frac{1}{f}$$

Fabry-Perot Transmission and Reflection (general; with gain or absorption):

$$T(\theta,G_0) = \frac{G_0(1-R_1)(1-R_2)}{\left(1-G_0\sqrt{R_1R_2}\right)^2 + 4G_0\sqrt{R_1R_2}\sin^2(\theta)}$$

$$R(\theta,G_0) = \frac{\left(\sqrt{R_1} - G_0\sqrt{R_2}\right)^2 + 4G_0\sqrt{R_1R_2}\sin^2(\theta)}{\left(1-G_0\sqrt{R_1R_2}\right)^2 + 4G_0\sqrt{R_1R_2}\sin^2(\theta)}$$

$$2\Delta\theta_{1/2} = \frac{1-G_0\sqrt{R_1R_2}}{G_0^{1/2}\sqrt[4]{R_1R_2}}$$

$$\theta = kd = \frac{\omega n d}{c}$$

for plane waves

Finesse $F = \frac{\pi\sqrt[4]{R_1R_2}}{1-\sqrt{R_1R_2}} = \frac{\Delta\nu_{FSR}}{\Delta\nu_{1/2}}$

Free Spectral Range: $\Delta\nu_{FSR} = \frac{c}{2nd} = \frac{1}{\tau_{RT}}$

Photon Lifetime:

$$\tau_p = \frac{\tau_{RT}}{1-R_1R_{21}} \approx \frac{1}{2\pi\Delta\nu_{1/2}}$$

General Resonance Condition:
roundtrip phase change= q2π

Blackbody Radiation (energy density): $\rho(\nu)d\nu = \frac{8\pi n^3 h\nu^3 d\nu}{c^3}\frac{1}{e^{h\nu/kT}-1}$

Lorentzian line shape:
e.g. in natural or pressure broadened

$$g(\nu) = \frac{\Delta\nu_h/2\pi}{(\nu-\nu_0)^2 + (\Delta\nu_h/2)^2}$$

Doppler broadened line shape

$$g(\nu) = \left(\frac{4\ln 2}{\pi}\right)^{1/2}\frac{1}{\Delta\nu_D}\exp\left[(-4\ln 2)\left(\frac{\nu-\nu_0}{\Delta\nu_D}\right)^2\right] \quad \text{with}$$

$$\Delta\nu_D = \left(\frac{8kT\ln 2}{Mc^2}\right)^{1/2}\nu_0$$

Formula Sheet (page 2)
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Gain in a two-level system: $\gamma(\nu) = \sigma(\nu)\left[N_2 - \frac{g_2}{g_1}N_1\right]$ Gain cross section: $\sigma(\nu) = A_{21}\frac{\lambda^2}{8\pi n^2}g(\nu)$

Lineshape Normalization: $\int g(\nu)d\nu = 1$ Beer's Law: $\frac{1}{I}\frac{dI}{dz} = -\alpha(I) + \gamma(I)$

Gain or absorption saturation in a homogenously-broadened system:

$\gamma(I) = \frac{\gamma_0}{1 + I/I_s}$ or $\alpha(I) = \frac{\alpha_0}{1 + I/I_s}$ $I_s(\nu) = \frac{h\nu}{\sigma(\nu)\tau_2}$

Einstein's relation: $\frac{A_{21}}{B_{21}} = \frac{8\pi n^3 h\nu^3}{c^3}$ $g_2 B_{21} = g_1 B_{12}$ $\frac{N_2}{N_1} = \frac{g_2}{g_1}e^{-(E_2-E_1)/kT}$

Degeneracy factors of level i: $g_i = 2J_i + 1$ (J_i is total angular momentum quantum number of that level)

Laser amplifier gain: $\ln\frac{G}{G_0} + \frac{G-1}{I_s/I_{in}} = 0$ where $G_0 = \exp(\gamma_0 L_g)$ is the small-signal gain, $G = I_{out}/I_{in}$

ABCD Matrices $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$ AD-BC=1 $\begin{pmatrix} r_2 \\ r_2' \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} r_1 \\ r_1' \end{pmatrix}$

Free space of length d $\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$	Dielectric interface (from n_1 to n_2) $\begin{pmatrix} 1 & 0 \\ 0 & n_1/n_2 \end{pmatrix}$
Propagation in a medium of length d and index n_2=n immersed in vacuum (n_1=1). $\begin{pmatrix} 1 & d/n \\ 0 & 1 \end{pmatrix}$	Thin lens of focal length f $\begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$
Mirror with radius of curvature R $\begin{pmatrix} 1 & 0 \\ -2/R & 1 \end{pmatrix}$	Spherical dielectric interface $\begin{pmatrix} 1 & 0 \\ (1-n_1/n_2)/R & n_1/n_2 \end{pmatrix}$

ABCD rule for Gaussian beams:

$$q_2 = \frac{Aq_1 + B}{Cq_1 + D}$$ where

$$q(z) = z + iz_0$$
or
$$\frac{1}{q(z)} = \frac{1}{R(z)} - i\frac{\lambda_0}{\pi n w(z)^2}$$

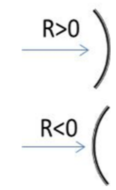
Stability condition: -1<(A+D)/2<1

Laser Threshold: $G_0^2 S = 1$

S= passive cavity survival factor (=$R_1 R_2$ for a simple two mirror cavity)

Photon Density (Photon Number per Volume) $\frac{N_p}{V} = \frac{I}{h\nu c/n_g}$

Convention for refractive and reflective surfaces:



Fundamental Physical Constants

Quantity	Symbol	Value
Speed of light	c	$2.99792458 \ x \ 10^{8} \ m/s$
Planck constant	h	$6.6260755 \ x \ 10^{-34} \ J \cdot s$
Planck constant	h	$4.1356692 \ x \ 10^{-15} \ eV \cdot s$
Planck hbar	$\hbar$	$1.0545727 \ x \ 10^{-34} \ J \cdot s$
Planck hbar	$\hbar$	$6.582121 \ x \ 10^{-16} \ eV \cdot s$
Gravitation constant	G	$6.67259 \ x \ 10^{-11} \ m^{3} \cdot kg^{-1} \cdot s^{-2}$
Boltzmann constant	k	$1.380658 \ x \ 10^{-23} \ J/K$
Molar gas constant	R	$8.314510 \ J/mol \cdot K$
Charge of electron	e	$1.60217733 \ x \ 10^{-19} \ C$
Permeability of vacuum	μ_0	$4\pi \ x \ 10^{-7} \ N/A^{2}$
Permittivity of vacuum	ε_0	$8.854187817 \ x \ 10^{-12} \ F/m$
Mass of electron	m_e	$9.1093897 \ x \ 10^{-31} \ kg$
Mass of proton	m_p	$1.6726231 \ x \ 10^{-27} \ kg$
Mass of neutron	m_n	$1.6749286 \ x \ 10^{-27} \ kg$
Avogadro's number	N_A	$6.0221367 \ x \ 10^{23} \ /mol$
Stefan-Boltzmann constant	σ	$5.67051 \ x \ 10^{-8} \ W/m^{2} \cdot K^{4}$
Rydberg constant	R_∞	$10973731.534 \ m^{-1}$
Bohr magneton	μ_B	$9.2740154 \ x \ 10^{-24} \ J/T$
Bohr radius	a_0	$0.529177249 \ x \ 10^{-10} m$
Standard atmosphere	atm	$101325 \ Pa$

1G = 10^{-4} T , 1 eV = 1.602 × 10^{-19} J, 1 dyne = 10^{-5} N, 1 erg = 10^{-7} J