Laser Physics I (PHYC/ECE 464)
FALL 2010

Midterm Exam, Closed Book, Closed Notes

Time: 5:30 – 6:45 pm

NAME ....
last *first*

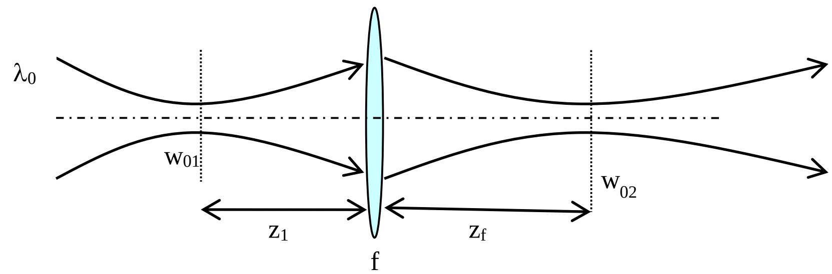
Score

Total= 100 points (+10 Bonus points)

Please staple and return these pages with your exam.

Instructor: M. Sheik-Bahae

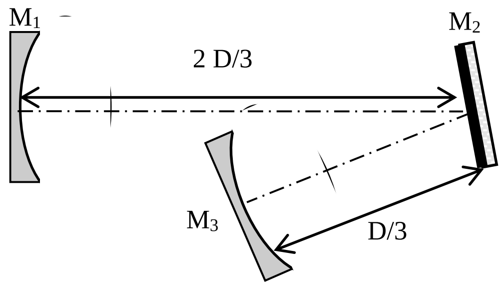
1. A laser beam with wavelength λ_0 is incident from left on a lens (f) placed at a distance z_1 after its minimum waist. ***(30 points)***



a) What is the **q**, beam radius **w** and radius of curvature **R** at the following locations:
   - a.1 Just before the lens?
   - a.2 Right after the lens?
   - a.3 At an arbitrary distance z_2 after the lens?

b) Show how to determine the location (z_f) and the value (w_{02}) of the new focus in terms of the given parameters.

2. ***(20 points)*** Consider the laser cavity (shown below) consisting of two concave mirrors (both with radius **R**) and one flat mirror.

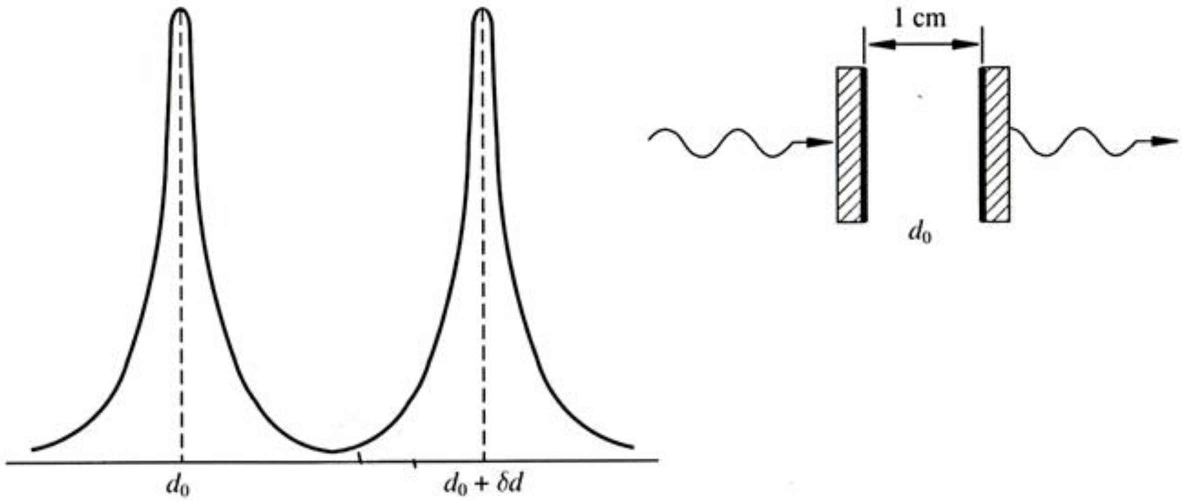


(a) Derive the stability condition.

(b) Find the location and the value of the minimum beam waist w_0.

3. ***(25 points)***

Drawn to scale on the graph below is the relative power transmission through a Fabry-Perot cavity when the distance d is increased slightly. The source is a He:Ne laser at $\lambda_0 =$ ~~6328 Å~~. 1 μm



(a) What is the distance δd?
(b) What is the finesse of the cavity?
(c) What is the cavity Q?

(Note: this was a HW problem)

4. ***(25 points)*** Consider the ring laser system with its homogenous lineshape shown below. The following parameters are known:

$A_{21}=2.5\times10^{6}\ s^{-1}$,

n(gain medium)=1.5,

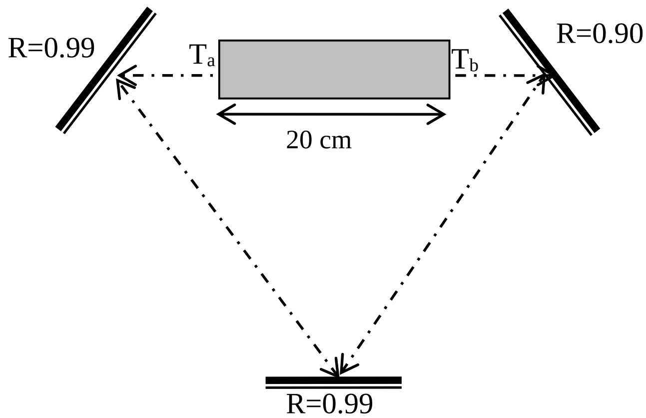
upper state lifetime $(\tau_2)=10$ *ns, and*

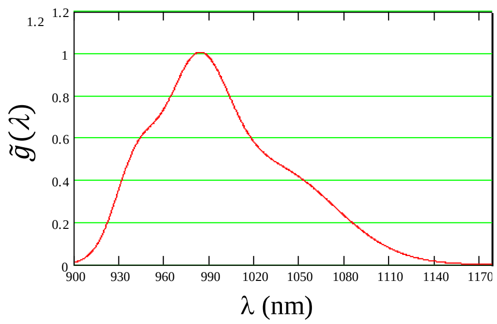
$T_a=T_b=0.995\%$ $(g_1=g_2=1)$

Use the given information to estimate:

(a) The stimulated emission cross section (σ) and the saturation intensity (I_s) at 1020 nm.

(b) The laser threshold population inversion $(N_2\text{-}N_1)_{th}$.





Bonus (10 points): If the cavity round-trip time is 10 ns, *estimate* the number of longitudinal modes that will initially experience gain if pumped 2 times above the laser threshold.

Formula Sheet
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Hermite-Gaussian Beams:

$$\frac{E(x,y,z)}{E_0} = H_m\left(\frac{\sqrt{2}x}{w(z)}\right)H_p\left(\frac{\sqrt{2}y}{w(z)}\right)\frac{w_0}{w(z)}\exp\left(-i\frac{kr^2}{2q(z)}\right)\times\exp\left(-i\left[kz-(1+m+p)\tan^{-1}(z/z0)\right]\right)$$

$$\frac{1}{q(z)} = \frac{1}{R(z)} - i\frac{\lambda}{\pi w^2(z)}, \qquad w^2(z) = w_0^2\left(1+\frac{z^2}{z_0^2}\right), \qquad R(z) = z\left(1+\frac{z_0^2}{z^2}\right), \qquad z_0 = \frac{\pi n w_0^2}{\lambda_0}$$

$$k = n\frac{\omega}{c} = \frac{2\pi n}{\lambda_0}$$

Irradiance: $I = \langle S\rangle = \frac{nc\varepsilon_0}{2}E_0^2$

Snell's Law: $n_i\sin\theta_i = n_t\sin\theta_t$

Fresnel:

$$r_\parallel = \frac{n_t\cos\theta_i - n_i\cos\theta_t}{n_t\cos\theta_i + n_i\cos\theta_t} = \frac{\tan(\theta_i-\theta_t)}{\tan(\theta_i+\theta_t)} \qquad r_\perp = -\frac{n_i\cos\theta_i - n_t\cos\theta_t}{n_i\cos\theta_i + n_t\cos\theta_t} = -\frac{\sin(\theta_i-\theta_t)}{\sin(\theta_i+\theta_t)}$$

$$t_\parallel = \frac{2n_i\cos\theta_i}{n_i\cos\theta_t + n_t\cos\theta_i} = \frac{2\sin\theta_t\cos\theta_i}{\sin(\theta_i+\theta_t)\cos(\theta_i-\theta_t)} \qquad t_\perp = \frac{2n_i\cos\theta_i}{n_i\cos\theta_i + n_t\cos\theta_t} = \frac{2\sin\theta_t\cos\theta_i}{\sin(\theta_i+\theta_t)}$$

when there is total internal reflection at an air or vacuum interface:

$$r_\parallel = \frac{\cos\theta_i - in_i\sqrt{n_i^2\sin^2\theta_i - 1}}{\cos\theta_i + in_i\sqrt{n_i^2\sin^2\theta_i - 1}} \qquad r_\perp = \frac{n_i\cos\theta_i - i\sqrt{n_i^2\sin^2\theta_i - 1}}{n_i\cos\theta_i + i\sqrt{n_i^2\sin^2\theta_i - 1}}$$

Lens-maker's formula:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Lens Transformation of a Gaussian beam:

$$\frac{1}{R_{out}} = \frac{1}{R_{in}} - \frac{1}{f}$$

Fabry-Perot Transmission and Reflection (general; with gain or absorption):

$$T(\theta,G_0) = \frac{G_0(1-R_1)(1-R_2)}{\left(1-G_0\sqrt{R_1R_2}\right)^2 + 4G_0\sqrt{R_1R_2}\sin^2(\theta)}$$

$$R(\theta,G_0) = \frac{\left(\sqrt{R_1}-\sqrt{R_2}\right)^2 + 4G_0\sqrt{R_1R_2}\sin^2(\theta)}{\left(1-G_0\sqrt{R_1R_2}\right)^2 + 4G_0\sqrt{R_1R_2}\sin^2(\theta)}$$

$$2\Delta\theta_{1/2} = \frac{1-G_0\sqrt{R_1R_2}}{G_0^{1/2}\sqrt[4]{R_1R_2}}$$

$$\theta = kd = \frac{\omega nd}{c}$$

for plane waves

Finesse $F = \frac{\pi\sqrt[4]{R_1R_2}}{1-\sqrt{R_1R_2}} = \frac{\Delta\nu_{FSR}}{\Delta\nu_{1/2}}$

Free Spectral Range: $\Delta\nu_{FSR} = \frac{c}{2nd} = \frac{1}{\tau_{RT}}$

Photon Lifetime: $\tau_p = \frac{\tau_{RT}}{1-R_1R_2} \approx \frac{1}{2\pi\Delta\nu_{1/2}}$

General Resonance Condition:
roundtrip phase change= q2π

Blackbody Radiation (energy density): $\rho(\nu)d\nu = \frac{8\pi n^3 h\nu^3 d\nu}{c^3}\frac{1}{e^{h\nu/kT}-1}$

Lorentzian line shape:
e.g. in natural or pressure broadened

$$g(\nu) = \frac{\Delta\nu_h/2\pi}{(\nu-\nu_0)^2 + (\Delta\nu_h/2)^2}$$

Doppler broadened line shape

$$g(\nu) = \left(\frac{4\ln 2}{\pi}\right)^{1/2}\frac{1}{\Delta\nu_D}\exp\left[(-4\ln 2)\left(\frac{\nu-\nu_0}{\Delta\nu_D}\right)^2\right]$$ with

$$\Delta\nu_D = \left(\frac{8kT\ln 2}{Mc^2}\right)^{1/2}\nu_0$$

Formula Sheet (page 2)
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Gain in a two-level system: $\gamma(\nu) = \sigma(\nu)\left[N_2 - \frac{g_2}{g_1}N_1\right]$ Gain cross section: $\sigma(\nu) = A_{21}\frac{\lambda^2}{8\pi n^2}g(\nu)$

Lineshape Normalization: $\int g(\nu)d\nu = 1$ Beer's Law: $\frac{1}{I}\frac{dI}{dz} = -\alpha(I) + \gamma(I)$

Gain or absorption saturation in a homogenously-broadened system:

$\gamma(I) = \frac{\gamma_0}{1 + I/I_s}$ or $\alpha(I) = \frac{\alpha_0}{1 + I/I_s}$ $I_s(\nu) = \frac{h\nu}{\sigma(\nu)\tau_2}$

Einstein's relation: $\frac{A_{21}}{B_{21}} = \frac{8\pi n^3 h\nu^3}{c^3}$ $g_2 B_{21} = g_1 B_{12}$ $\frac{N_2}{N_1} = \frac{g_2}{g_1}e^{-(E_2-E_1)/kT}$

Laser amplifier gain: $\ln\frac{G}{G_0} + \frac{G-1}{I_s/I_{in}} = 0$ where $G_0=\exp(\gamma_0 L_g)$ is the small-signal gain, $G=I_{out}/I_{in}$

ABCD Matrices $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$ AD-BC=1 $\begin{pmatrix} r_2 \\ r_2' \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} r_1 \\ r_1' \end{pmatrix}$

ABCD rule for Gaussian Beams

$$q_2 = \frac{Aq_1 + B}{Cq_1 + D}$$

where

$q(z) = z + iz_0$

or

$$\frac{1}{q(z)} = \frac{1}{R(z)} - i\frac{\lambda_0}{\pi n w(z)^2}$$

Gaussian pulse propagation (broadening) in dispersive media

$$\tau_p^2(z) = \tau_{p0}^2\left(1 + \frac{z^2}{\ell_0^2}\right)$$ where

dispersion length $\ell_0 = \frac{\tau_{p0}^2}{2|\beta_2|}$ and

group velocity dispersion (GVD)

$$\beta_2 = \frac{\lambda^3}{2\pi c^2}\frac{d^2 n}{d\lambda^2} = \frac{\lambda^2}{2\pi c}D$$

Free space of length d $\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$	Dielectric interface (from n_1 to n_2) $\begin{pmatrix} 1 & 0 \\ 0 & n_1/n_2 \end{pmatrix}$
Propagation in a medium of length d and index n_2=n immersed in vacuum (n_1=1). $\begin{pmatrix} 1 & d/n \\ 0 & 1 \end{pmatrix}$	Thin lens of focal length f $\begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$

Mirror with radius of curvature R	Spherical dielectric interface
$\begin{pmatrix} 1 & 0 \\ -2/R & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ (1-n_1/n_2)/R & n_1/n_2 \end{pmatrix}$

Photon Density (Photon Number per Volume) $\frac{N_p}{V} = \frac{I}{h\nu c/n_g}$

$\frac{dN_p}{dt} = \frac{G^2 S - 1}{\tau_{RT}} N_p + N_2 c\sigma$ (Photon number growth dynamics due to *stimulated* and *spontaneous* emission processes)

S (survival factor)=$R_1 R_2$ for a simple two mirror cavity, G^2=roundtrip gain

Schawlow-Townes limit for laser linewidth: $\Delta\nu_{osc} \approx 2\pi \frac{h\nu}{P_{out}} \left(\Delta\nu_{1/2}\right)^2$

Fundamental Physical Constants

Quantity	Symbol	Value
Speed of light	c	$2.99792458 \; x \; 10^{8} \; m/s$
Planck constant	h	$6.6260755 \; x \; 10^{-34} \; J \cdot s$
Planck constant	h	$4.1356692 \; x \; 10^{-15} \; eV \cdot s$
Planck hbar	$\hbar$	$1.0545727 \; x \; 10^{-34} \; J \cdot s$
Planck hbar	$\hbar$	$6.582121 \; x \; 10^{-16} \; eV \cdot s$
Gravitation constant	G	$6.67259 \; x \; 10^{-11} \; m^{3} \cdot kg^{-1} \cdot s^{-2}$
Boltzmann constant	k	$1.380658 \; x \; 10^{-23} \; J/K$
Molar gas constant	R	$8.314510 \; J/mol \cdot K$
Charge of electron	e	$1.60217733 \; x \; 10^{-19} \; C$
Permeability of vacuum	μ_0	$4\pi \; x \; 10^{-7} \; N/A^{2}$
Permittivity of vacuum	ε_0	$8.854187817 \; x \; 10^{-12} \; F/m$
Mass of electron	m_e	$9.1093897 \; x \; 10^{-31} \; kg$
Mass of proton	m_p	$1.6726231 \; x \; 10^{-27} \; kg$
Mass of neutron	m_n	$1.6749286 \; x \; 10^{-27} \; kg$
Avogadro's number	N_A	$6.0221367 \; x \; 10^{23} \; /mol$
Stefan-Boltzmann constant	σ	$5.67051 \; x \; 10^{-8} \; W/m^{2} \cdot K^{4}$
Rydberg constant	R_∞	$10973731.534 \; m^{-1}$
Bohr magneton	μ_B	$9.2740154 \; x \; 10^{-24} \; J/T$
Bohr radius	a_0	$0.529177249 \; x \; 10^{-10} m$
Standard atmosphere	atm	$101325 \; Pa$

1G = 10^{-4} T , 1 eV = 1.602 × 10^{-19} J, 1 dyne = 10^{-5} N, 1 erg = 10^{-7} J