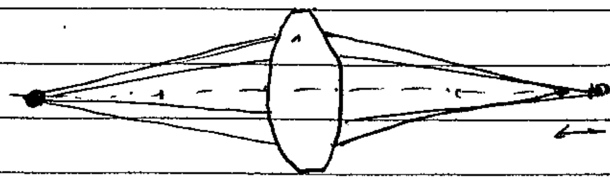
PHYC/ECE 463 Advanced Optics I
Fall 2007
Homework #8, Due Wednesday Oct. 17

1-Problem 4.26 (K&F)

2-Problem 4.38 (K&F)

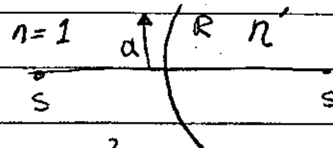
Reminder: Midterm Exam, Wednesday Oct. 24

4 KF. 4.26



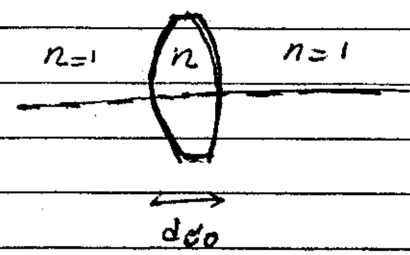
* Solution (1)

Start with the expression derived (in class) for a single refracting surface



$$\frac{n}{s} + \frac{n'}{s'} = \frac{n'-n}{R} + \frac{a^2}{2}\left[\frac{n}{s}\left(\frac{1}{s}+\frac{1}{R}\right)^2 + \frac{n'}{s'}\left(\frac{1}{s'}-\frac{1}{R}\right)^2\right]$$

a thin lens formed by two refracting surfaces of curvatures R and $-R$ can now be analysed. The image from the first surface (say at S_i) will act as the object for the second surface.



First surface $$\frac{1}{S} + \frac{n}{S_i} = \frac{n-1}{R} + \frac{a^2}{2}\left[\frac{1}{S}\left(\frac{1}{S}+\frac{1}{R}\right)^2 + \frac{n}{S_i}\left(\frac{1}{S_i}-\frac{1}{R}\right)^2\right] \quad (1)$$

Second " $$-\frac{n}{S_i} + \frac{1}{S'} = \frac{-n+1}{-R} + \frac{a^2}{2}\left[-\frac{n}{S_i}\left(\frac{1}{S_i}-\frac{1}{R}\right)^2 + \frac{1}{S'}\left(\frac{1}{S'}+\frac{1}{R}\right)^2\right] \quad (2)$$

We must now eliminate S_i. First add (1) + (2)

$$\frac{1}{S} + \frac{1}{S'} = \frac{2(n-1)}{R} + \frac{a^2}{2}\left[\frac{1}{S}\left(\frac{1}{S}+\frac{1}{R}\right)^2 + \frac{1}{S'}\left(\frac{1}{S'}+\frac{1}{R}\right)^2 - \frac{4n}{S_i}\frac{1}{S_i R}\right] \quad ($$

Since we are interested in aberrations functions to the lowest order in a^2, we can replace $\frac{1}{s_i}$ in equation (3) from eqn. 1 (ignoring a^2 term). Thus

$$\frac{n}{s_i} \approx \frac{n-1}{R} - \frac{1}{s} \quad \Rightarrow \quad \frac{n}{s_i^2} = \frac{1}{n}\left(\frac{n-1}{R} - \frac{1}{s}\right)^2$$

Now eqn. (3) can be rewritten as

$$\frac{1}{s} + \frac{1}{s'} = \frac{2(n-1)}{R} + \frac{a^2}{2}\left(\frac{1}{s}\left(\frac{1}{s}+\frac{1}{R}\right)^2 + \frac{1}{s'}\left(\frac{1}{s'}+\frac{1}{R}\right)^2 - \frac{4}{nR}\left(\frac{n-1}{R}-\frac{1}{s}\right)^2\right)$$

Consider two cases:

* Paraxial ($a \cong 0$) $\qquad \frac{1}{s} + \frac{1}{s'} = \frac{2(n-1)}{R} = \frac{1}{f}$

* non paraxial $\qquad \frac{1}{s} + \frac{1}{s'+\Delta s'} = \frac{1}{f} + \frac{a^2}{2}\left(\qquad \right)$

$$\frac{1}{s'+\Delta s'} \approx \frac{1}{s'} - \frac{\Delta s'}{s'^2}$$

Subtract the two eqns.

$$\Rightarrow \frac{\Delta s'}{s'^2} \approx -\frac{a^2}{2}\left[\frac{1}{s}\left(\frac{1}{s}+\frac{1}{2(n-1)f}\right)^2 + \left(\frac{1}{f}-\frac{1}{s}\right)\left(\frac{2n-1}{2(n-1)}\times\frac{1}{f}-\frac{1}{s}\right)^2 - \frac{2}{n(n-1)f}\left(\frac{1}{2f}-\frac{1}{s}\right)^2\right]$$

$$\boxed{\frac{\Delta s'}{f} = -\frac{a^2}{2}\times\frac{f s^2}{(f-s)^2}\left[\frac{1}{s}\left(\frac{1}{s}+\frac{1}{2(n-1)f}\right)^2 + \left(\frac{1}{f}-\frac{1}{s}\right)\left(\frac{2n-1}{2(n-1)f}-\frac{1}{s}\right)^2 - \frac{2}{n(n-1)f}\left(\frac{1}{2f}-\frac{1}{s}\right)^2\right]}$$

Now let $s = 2f$

$$\frac{\Delta s'}{f} \approx -\frac{a^2}{2}\times\frac{4f^3}{f^2}\left[\frac{1}{2f}\left(\frac{1}{2f}+\frac{1}{2(n-1)f}\right)^2 + \frac{1}{2f}\left(\frac{2n-1}{2(n-1)}\frac{1}{f}-\frac{1}{2f}\right)^2 - 0\right]$$

$$\frac{\Delta s'}{f} \simeq -\frac{a^2}{2}\times\frac{1}{f^2}\frac{n^2}{(n-1)^2} \quad \Rightarrow \quad \boxed{\frac{\Delta s'}{f} = -\frac{1}{2}\frac{a^2}{f^2}\frac{n^2}{(n-1)^2}}$$

#4

* Solution #2 :

Use the expression derived in K&F, (Eqn. 4-91, pp24

$$\frac{\Delta S'}{f} = 4\, {}_0C_{40}\, f^3 \left(\frac{\tilde{r}}{f}\right)^2 \left(\frac{S'}{f}\right)^2$$

$$\tilde{r} = a \qquad S' = \frac{fS}{S-f} = 2f$$

$$\frac{\Delta S'}{f} = 16\, {}_0C_{40}\, f^3 \left(\frac{a}{f}\right)^2 \qquad (1)$$

${}_0C_{40}$ is given by Eqn. 4-88 (K&F) as a function of q and p parameters where,

$$q = \frac{R'+R}{R'-R} = 0 \quad \text{for symmetric lens}$$

$$p = 1 - \frac{2f}{S} = 0 \quad \text{for } S = 2f$$

Thus

$$ {}_0C_{40} = -\frac{1}{32 f^3 n(n-1)} \times \frac{n^3}{n-1} \qquad (2)$$

And replacing ${}_0C_{40}$ from (2) into (1)

$$\boxed{\frac{\Delta S'}{f} = -\frac{1}{2}\left(\frac{a}{f}\right)^2 \frac{n^2}{(n-1)^2}}$$

4.38

Glass Crown $510:635 \Rightarrow n_{d1} = 1.510 \quad V_1 = 63.5$

Flint

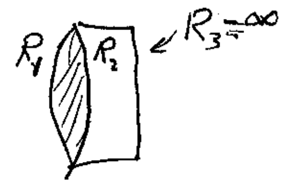
$620:364 \Rightarrow n_{d2} = 1.620 \quad V_2 = 36.4$

$$f_1 V_1 + f_2 V_2 = 0. \qquad \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f} = \frac{1}{100\text{ cm}}$$

$$\Rightarrow \frac{1}{f_1} = \frac{-V_1}{V_2 - V_1}\frac{1}{f} \qquad \frac{1}{f_2} = \frac{V_2}{V_2 - V_1}\frac{1}{f}$$

$$\frac{1}{f_1} = \frac{-63.5}{36.4 - 63.5} \times \frac{1}{f} \qquad \frac{1}{f_2} = \frac{36.4}{36.4 - 63.5}\frac{1}{f}$$

$$\frac{1}{f_1} = 2.343 \times \frac{1}{f} \qquad \frac{1}{f_2} = -1.343 \times \frac{1}{f}$$



$$\frac{1}{f_1} = \frac{2.343}{100} = (1.510 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right). \Rightarrow \frac{1}{R_1} - \frac{1}{R_2} = 0.0459$$

$$\frac{1}{f_2} = -\frac{1.343}{100} = (1.620 - 1)\left(\frac{1}{R_2} - \frac{1}{\infty}\right) = \qquad \frac{1}{R_2} = -0.0216$$

$R_2 = -46.16$ cm

$R_1 = 41.25$ cm.