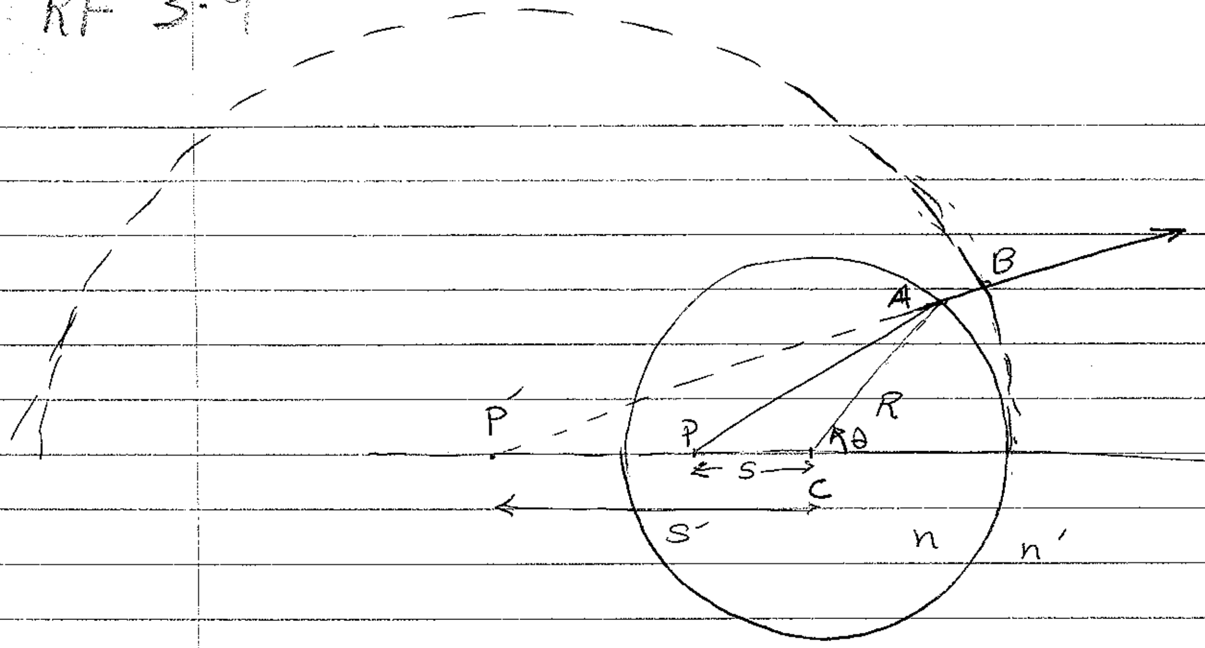
KF 3.9



For P' to be a virtual image of P, the O.P.L given by $n\overline{PA} + n'\overline{AB}$ must be stationary for every point A on the circle (Fermat principal). The point B describes a circle centered at P' with a radius: more ... or equal to $S'+R$. Here: $R' = S'+R$

$$O.P.L. = n\,\overline{PA} + n'\,\overline{AB} = n\,\overline{PA} + n'(R' - \overline{AP'})$$

From Fig:

$$\overline{AP} = \sqrt{R^2 + S^2 + 2RS\cos\theta}$$
$$\overline{AP'} = \sqrt{R^2 + S'^2 + 2RS'\cos\theta}$$

Thus

$$O.P.L = n\cdot\sqrt{R^2+S^2+2RS\cos\theta} + n'(R+S') - n'\sqrt{R^2+S'^2+2RS'\cos\theta}$$

Fermat Principal:

$$\frac{d\,O.P.L.}{d\theta} = \frac{-nRS\sin\theta}{\sqrt{R^2+S^2+2RS\cos\theta}} + \frac{n'RS'\sin\theta}{\sqrt{R^2+S'^2+2RS'\cos\theta}} = 0$$

$$n^2S^2(R^2+S'^2+2RS'\cos\theta) = n'^2S'^2(R^2+S^2+2RS\cos\theta) \qquad \text{for all } \theta$$

$$2RSS'\cos\theta\,(n^2 S - n'^2 S') + n^2S^2(R^2+S'^2) - n'^2S'^2(R^2+S^2) = 0$$

$$n^2 S - n'^2 S' = 0 \qquad \boxed{n^2 S = n'^2 S'}$$

$$n^2 S^2(R^2+S'^2) - n'^2 S'^2(R^2+S^2) = 0$$

use $n^2 S = n'^2 S'$

$$S(R^2+S'^2) - S'(R^2+S^2) = 0 \Rightarrow R^2(S-S') - SS'(S-S') = 0$$

$$(S-S')(R^2-SS') = 0 \qquad \boxed{R^2 = SS'}$$

$$SS' = R^2$$

multiply by $n^2 n'^2$

$$n^2 S\, n'^2 S' = n^2 n'^2 R^2$$

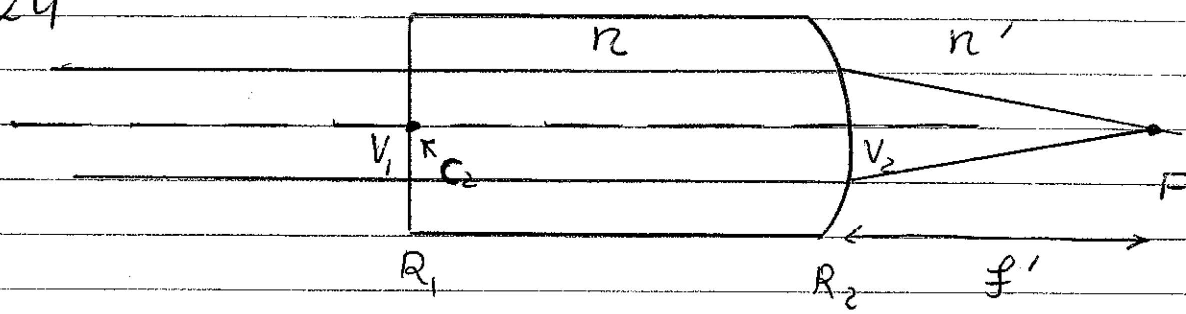
$$n^2 S \times n^2 S = (nn'R)^2$$

or

$$\boxed{n^2 S = nn'R = n'^2 S'}$$

#2

KF 3.24

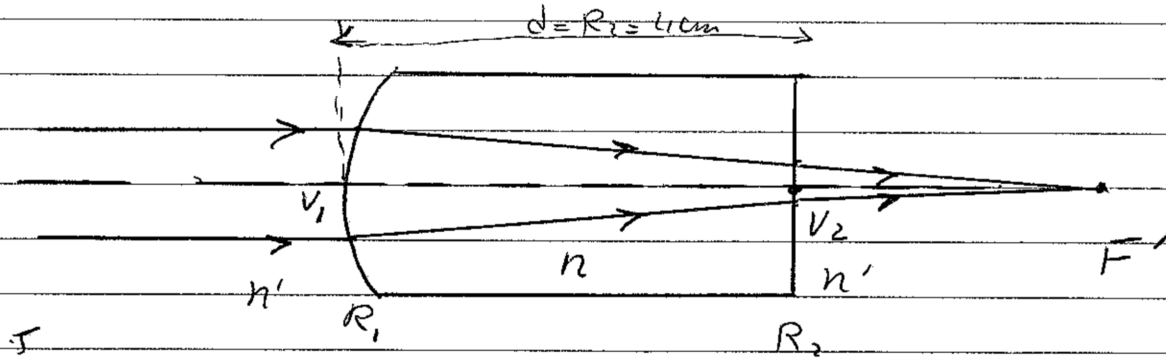


$$\begin{cases} n = 1.5 \\ R_1 = \infty \quad \text{flat} \\ R_2 = -4 \text{ cm} \\ n' = 1 \\ f' = ? \end{cases}$$

(a) Since $R_1 = \infty$ and incident rays are parallel ($\alpha_1 = 0$) we only consider the refraction at the 2nd surface

$$f' = \frac{n' R_2}{n' - n} = \frac{1 \times (-4)}{1 - 1.5} = 8 \text{ cm}$$

(b)



$$\begin{cases} n = 1.5 \\ n' = 1 \\ R_1 = +4 \text{ cm} \\ R_2 = \infty \\ d = |R_2| = 4 \text{ cm} \end{cases}$$

incident ray : $\begin{pmatrix} n\alpha_1 = 0 \\ x_1 \end{pmatrix}$

Consider the following transformation:

$\mathcal{R}_1$: (1st surface refraction)

$\mathcal{T}_{12}$: (translation through glass)

$\mathcal{R}_2$: (2nd surface refraction)

$\mathcal{T}_{23}$: (Translation to the focus F')

Applying R_1

$$R_1 \begin{cases} X'_1 = X_1 \\ \alpha'_1 = \cancel{\frac{n'}{n}\alpha_1} + \frac{(n'-n)X_1}{n R_1} \end{cases}$$

$$T_{12} \begin{cases} \alpha_2 = \alpha'_1 = \frac{(n'-n)X_1}{n R_1} \\ X_2 = X'_1 + d\,\alpha'_1 = X_1 + R_1 \frac{(n'-n)}{n R_1} X_1 = \frac{n'}{n} X_1 \end{cases}$$

$$R_2 \begin{cases} X'_2 = X_2 = \frac{n'}{n} X_1 \\ \alpha'_2 = \frac{n}{n'}\alpha_2 = \frac{n'-n}{n'} \frac{X_1}{R_1} \end{cases}$$

$$T_{23} \begin{cases} X_3 = X'_2 + \alpha'_2 f' = \frac{n'}{n} X_1 + \frac{n'-n}{n'} \frac{X_1}{R_1} f' \equiv 0 \\ \alpha_3 = \alpha'_2 = \frac{n'-n}{n'} \frac{X_1}{R_1} \end{cases}$$

$\left(\begin{array}{c}\text{focus} \\ \text{Condition} \\ X_3 = 0\end{array}\right)$

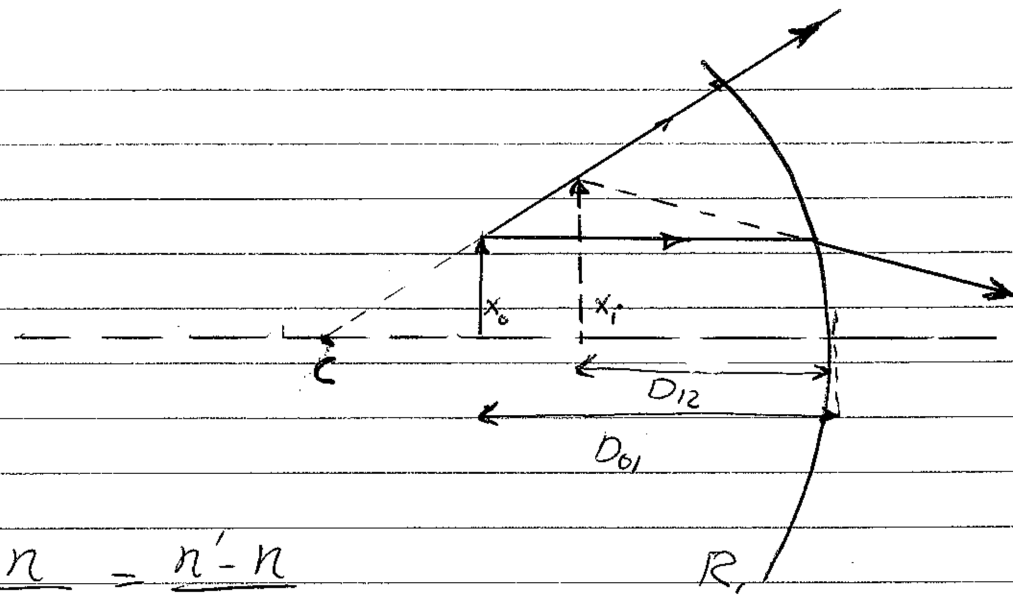
from the result of last transformation (T_{23}), independent of and X_1 value, $X_3 \equiv 0$ if:

$$f' = \frac{n'^2}{n(n-n')} R_1 = \frac{4}{1.5(1.5-1)} = \frac{8}{1.5} \text{ cm}$$

#3

KF. 3.27

$$\begin{cases} m_x = +1.2 \\ n = 1.65 \\ n' = 1 \\ D_{01} = 1.5\ cm \\ R_1 = ? \end{cases}$$



(1) $\rightarrow$ $$\frac{n'}{D_{12}} + \frac{n}{D_{01}} = \frac{n'-n}{R_1}$$

also $$m_x = -\frac{n}{n'}\frac{D_{12}}{D_{01}} \Rightarrow \frac{n'}{D_{12}} = -\frac{n}{D_{01}} \times \frac{1}{m_x} \leftarrow (2)$$

Sub. (2) into (1)

$$\frac{n}{D_{01}}\left(1 - \frac{1}{m_x}\right) = \frac{n'-n}{R_1}$$

$$R_1 = \frac{m_x}{m_x - 1} \times \frac{n'-n}{n} \times D_{01}$$

$$R_1 = \frac{1.2}{1.2-1} \times \frac{1-1.65}{1.65} \Rightarrow \boxed{R_1 = -3.54\ cm}$$