PHYC 463 Advanced Optics I
Fall 2007
Homework #3, Due Wednesday Sept. 12

1. Metal Optics

(a) Consider a material ($\tilde{n} = n - i\kappa$) with κ>>n. Show that reflectivity at normal incidence can be given by: *(2 points)*

$$R \approx 1 - \frac{4n}{1+\kappa^2}.$$

(b) Write down the expression $\tilde{n}^2$ for a metal (i.e. only N free electrons) assuming a finite collision time τ. (Ignore local field effects.) *(2 points)*

(c) In the visible to mid-infrared part of the spectrum we may take ωτ>>1. Show that for $\omega<\omega_p$, we can write: *(4 points)*

$$\kappa = \sqrt{\frac{\omega_p^2}{\omega^2} - 1} \quad \text{and} \quad n = \frac{1+\kappa^2}{\kappa} \times \frac{1}{\omega\tau}$$

(d) Show that reflectivity at normal incidence from a metal surface can be given by:

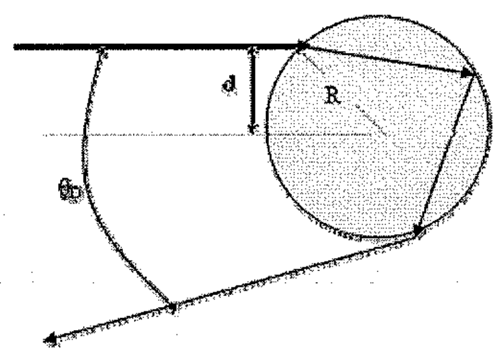
$$R \approx 1 - \frac{2\delta}{c\tau},$$

where δ=c/ωκ is the classical skin depth and c is the speed of light in vacuum. *(2 points)*

(e) For the metal described by Fig. 2.19 and its caption (K&F), calculate the normal incidence reflectivity for λ=600 nm and λ=400 nm. (2 points)

2. Snell's Law

A ray is incident on a dielectric sphere (radius *R* and refractive index *n*) at a distance *d* from the axis (as shown). Calculate the deviation angle θ_D for the exiting ray after one internal reflection. *(8 points)*



Monday 5:30

1-

(a) $$R = \frac{(n-1)^2 + k^2}{(n+1)^2 + k^2} = \frac{k^2+1+n^2-2n}{k^2+1+n^2-2n}$$

$$R \simeq \frac{1 - \frac{2n}{k^2+1}}{1 + \frac{2n}{k^2+1}} \simeq 1 - \frac{4n}{k^2+1}$$

(b) $$\tilde{n}^2 = 1 - \frac{\omega_p^2}{\omega^2 - i\frac{\omega}{\tau}} = (n - ik)^2$$

(c) $$\tilde{n}^2 = 1 - \frac{\omega_p^2/\omega^2}{1 + \frac{1}{\omega^2\tau^2}}\left(1 + \frac{i}{\omega\tau}\right)$$

$\omega\tau >> 1$

$$\tilde{n}^2 = 1 - \frac{\omega_p^2}{\omega^2} - i\,\frac{\omega_p^2/\omega^2}{\omega\tau}$$

$$\tilde{n} = \left[\left(1 - \frac{\omega_p^2}{\omega^2}\right)\left(1 - i\,\frac{\omega_p^2/\omega^2}{\left(\frac{\omega_p^2}{\omega^2} - 1\right)\omega\tau}\right)\right]^{1/2}$$

$$= \sqrt{1 + \frac{\omega_p^2}{\omega^2}} \times \left(1 + \frac{i\,\omega_p^2/\omega^2}{\left(\frac{\omega_p^2}{\omega^2} - 1\right)\omega\tau}\right)^{1/2}$$

$\omega < \omega_p$ and $\omega\tau >> 1$

$$= -i\sqrt{\frac{\omega_p^2}{\omega^2} - 1}\left(1 + i\,\frac{\omega_p^2}{\omega^2\, 2\left(\frac{\omega_p^2}{\omega^2} - 1\right)\omega\tau}\right)$$

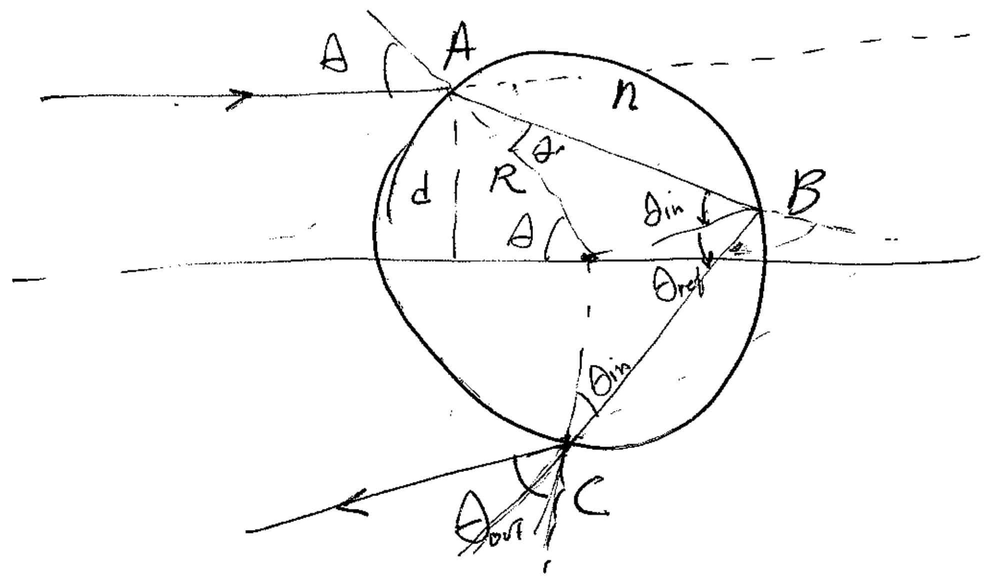
$$= -i\sqrt{\frac{\omega_p^2}{\omega^2} - 1} + \frac{\omega_p^2/\omega^2}{2\sqrt{\frac{\omega_p^2}{\omega^2} - 1}\;\omega\tau}$$

$$= -ik + \frac{k^2+1}{2k}\,\frac{1}{\omega\tau}$$

(d) Sub. into (a)

$$R = 1 - \frac{4 \times \frac{k^2+1}{2k} \times \frac{1}{\omega\tau}}{k^2+1} = 1 - \frac{4}{2k\omega\tau}$$

$$R = 1 - \frac{2\delta}{c\tau}$$



$$\sin\theta = \frac{d}{R} \qquad \sin\theta' = \frac{\sin\theta}{n} \qquad \theta_D^A = \theta - \theta'$$

@ point B $\qquad \theta_{in} = \theta' \qquad \theta_{ref} = \theta' \qquad \theta_D^B = \pi - 2\theta'$

@ point C $\qquad \theta_{in} = \theta' \Rightarrow \theta_{out} = \theta \qquad \theta_D^C = \theta - \theta'$

$$\theta_D^{Total} = \theta - \theta' + \pi - 2\theta' + \theta - \theta' = 2\theta - 4\theta' + \pi$$

$$\theta_D = \pi - \theta_D^{Total}$$

$$\theta_D = 2\left(- \sin^{-1}\frac{d}{R} + 2\sin^{-1}\frac{d}{nR}\right) = 4\sin^{-1}\frac{d}{nR} - 2\sin^{-1}\frac{d}{R}$$

$$\theta_D = 2\theta - 4\theta' + \pi$$