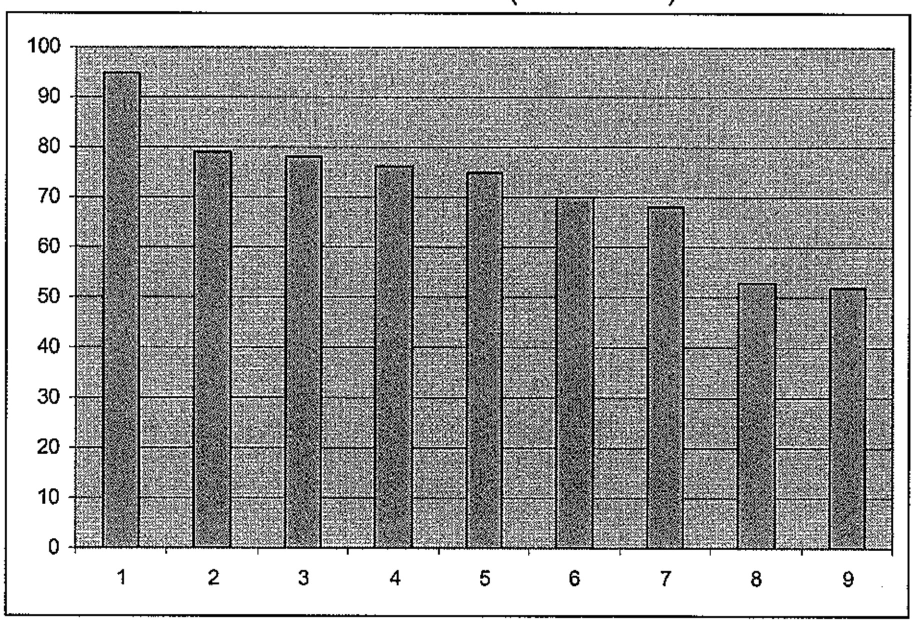
OPTICAL SCIENCE & ENGINEERING
University of New Mexico

PHYC/ECE 463 Advanced Optics I
Fall 2007
Midterm Exam, Closed Book, 1.5 hours

Class Score Chart (9 students)



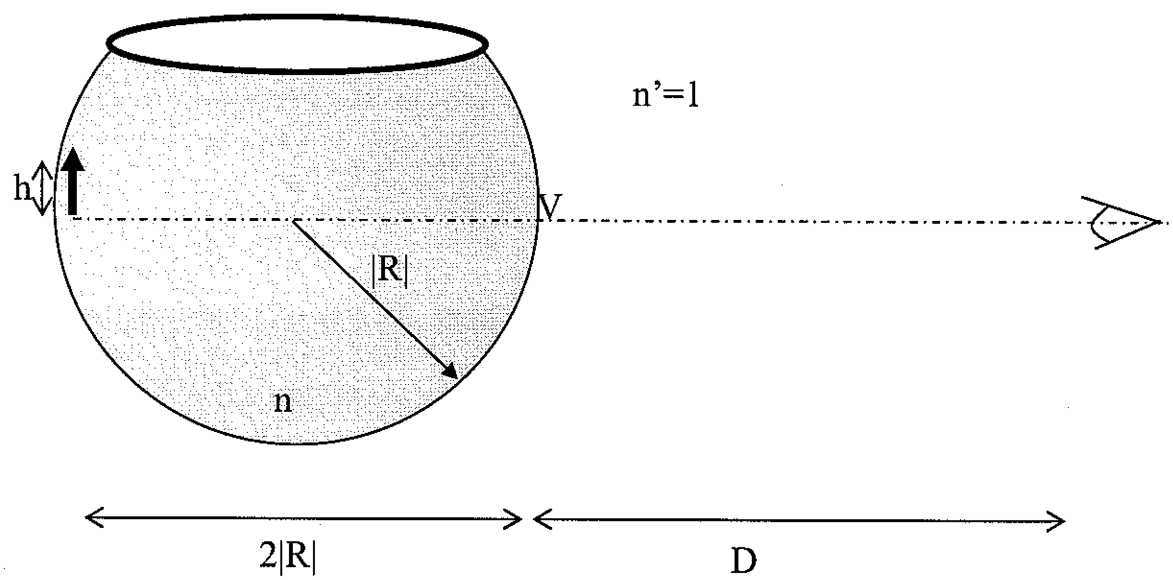
Average (with highest and lowest scores removed): 69

Problem 1. (35 points)
Consider the spherical fishbowl (radius $|R|$) filled with water having an index n. An object (e.g. a fish), indicated by the arrow, is located near the bowl wall (at a distance $2|R|$ from vertex V). Refraction due to thin glass walls can be ignored.

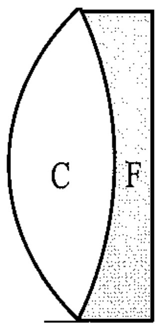
(a) Find the location of the image and discuss its real or imaginary nature with its dependence on n.

(b) Derive an expression for the angle α subtended by the (virtual) image as seen by an observer situated a distance D from V (as shown in the figure). The result should be in terms of n, $|R|$, D and h (the height of object). Assume small angle.

(c) Assume $|R|=10$ cm, and the bowl is filled with water ($n=1.3$). Find the distance D for which the observer sees the fish 25% larger than when it swims to the front side of the bowl (right behind vertex V).



Problem 2. (25 points) (same as HW 8)
Design an achromatic doublet with the contours indicated in the Fig. below. Use glass **510:635** as the crown component and the glass **620:364** as the flint component. Treat the lenses as thin lenses with zero separation and find a combination of radius of curvature that will produce a lens having a focal length of **100 cm**.



Note: Glass type is **xxx:yyy** where $xxx=(n_d-1)\times 1000$ and $yyy=10\times V$

Problem 3. (15 points)
Your are given the following four thin lenses with focal lengths *f* and diameter *D*:

Lens #	*f (cm)*	*D (cm)*	*Shape*
1	20	5	plano-convex
2	15	3	plano-convex
3	28	6	equi-convex
4	20	5	equi-convex

Which one would you use to light a match faster by focusing the sunlight? Explain.

Problem 4. (25 points)
Briefly (using less than 35 words for each) and using **diagrams**, **equations** (when necessary) **and examples** answer **only 2** of the following **3** questions.

(a) Describe the *spherical aberration. Give example for a thin lens.*

(b) Describe the *frustrated total internal reflection.*

(c) Why are metals highly reflective at certain wavelengths?

Formula Sheet PHYC/ECE 463 Advanced Optics I UNM/Fall 2007

harmonic plane wave: $E = \mathrm{Re}\{E_0 \exp(i\omega t - ik.r + \varphi)\}$ $\quad k = \frac{n\omega}{c} = \frac{2\pi n}{\lambda_0}$

Poynting vector $S = \frac{1}{\mu_0} E \times B$ Irradiance: $I = \langle S \rangle = \frac{\varepsilon_0 n c}{2} |E_0|^2$

$n_i \sin(\theta_i) = n_t \sin(\theta_t)$ *Snell's Law*

$$\rho_\pi = \frac{n_t \cos(\theta_i) - n_i \cos(\theta_t)}{n_t \cos(\theta_i) + n_i \cos(\theta_t)} = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}$$

$$\rho_\sigma = -\frac{n_i \cos(\theta_i) - n_t \cos(\theta_t)}{n_i \cos(\theta_i) + n_t \cos(\theta_t)} = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}$$

Fresnel's Reflectivity

$1 - \rho^2 = \tau\tau'$ $\quad R = |\rho|^2$

$R + T = 1$

If n is complex then $n \rightarrow \tilde{n} = n - i\kappa \equiv \sqrt{\varepsilon / \varepsilon_0} \equiv \sqrt{1 + \chi}$ in above expressions

Absorption coefficient (K or α) and skin depth (δ): $\alpha \equiv \frac{2}{\delta} = \frac{4\pi\kappa}{\lambda_0}$

Classical Electron Oscillator Model:

$$\chi = \frac{\omega_p^2}{\omega_0^2 - \omega^2 + i\omega/\tau} \quad \text{where } \omega_p = \sqrt{\frac{Nq^2}{m_0 \varepsilon_0}} \text{ (plasma frequency)}$$

Drude model for metals: $\omega_0 \rightarrow 0$

Group Velocity $v_g = \frac{d\omega}{dk}$

Light pressure (on perfecting absorbing surface) $P = \frac{I}{c}$

Prism with apex angle α at minimum deviation angle θ_D:

$$n = \frac{\sin\left(\frac{\alpha + \theta_D}{2}\right)}{\sin\left(\frac{\alpha}{2}\right)}$$

Numerical Aperture (NA) of an optical fiber: *f# (f-number)= f/D*

$$n_0 \sin(\theta_{\max}) = \sqrt{n_f^2 - n_c^2}$$

Lens-makers' formula: Gaussian imaging formula (thin lens) (refractive sphere)

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \qquad \frac{1}{f}=\frac{1}{S}+\frac{1}{S'} \qquad \frac{n'-n}{R}=\frac{n}{S}+\frac{n'}{S'}$$

$m_x=-\frac{S'}{S}$ (transverse magnification) $m_\alpha=\frac{n/n'}{m_x}$ (angular magnification)

Paraxial Ray Tracing Matrices:

Propagation $\begin{pmatrix}1 & 0\\ d/n & 1\end{pmatrix}$ (length d and index n)	Refractive surface $\begin{pmatrix}1 & -P\\ 0 & 1\end{pmatrix}$ *where P=(n'-n)/R*
Mirror with radius of curvature R $\begin{pmatrix}1 & -2/R\\ 0 & 1\end{pmatrix}$	Thin lens of focal length f $\begin{pmatrix}1 & -1/f\\ 0 & 1\end{pmatrix}$
Thick lens (Thickness D_l, index n_l) $\begin{pmatrix}1-\frac{P'D_l}{n_l} & -P-P'+\frac{PP'D_l}{n_l}\\ \frac{D_l}{n_l} & 1-\frac{PD_l}{n_l}\end{pmatrix}$ $P=(n_l-n)/R$, $P'=(n'-n_l)/R'$	Separated Doublet $\begin{pmatrix}1-\frac{d}{f_2 n_b} & -\frac{1}{f_1}-\frac{1}{f_2}+\frac{d}{f_2 f_1 n_b}\\ \frac{d}{n_b} & 1-\frac{d}{f_1 n_b}\end{pmatrix}$

between conjugate planes $\begin{pmatrix}m_\alpha\frac{n'}{n} & \tilde{M}_{12}\\ 0 & m_x\end{pmatrix}$	between principal planes $\begin{pmatrix}1 & M_{12}\\ 0 & 1\end{pmatrix}$
for telescopic Systems $\begin{pmatrix}m_\alpha\frac{n'}{n} & 0\\ M_{21} & m_x\end{pmatrix}$	position of principal planes $D=\left(\frac{n}{M_{12}}\right)(1-M_{11})$ $D'=\left(\frac{n'}{M_{12}}\right)(1-M_{22})$

Contact Doublet Achromatization:

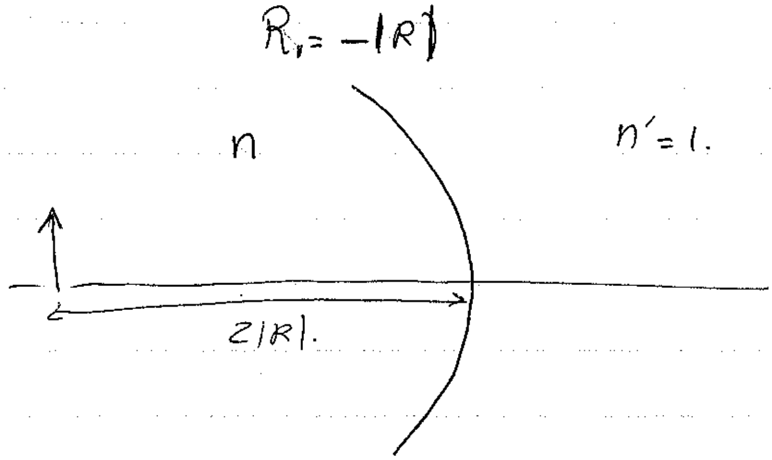
$f_1V_1+f_2V_2=0$ where $V=\frac{n_d-1}{n_f-n_c}$ is the Abbe number

Physical constants:

speed of light	$c\sim 2.998\times10^{8}$	m s^{-1}
electronic charge	$q\sim 1.602\times10^{-19}$	C
permittivity of vacuum	$\varepsilon_0\sim8.854\times10^{-12}$	F/m
electronic mass	$m_0\sim9.1094\times10^{-31}$	Kg
Planck constant	$h\sim6.626\times10^{-34}$	J.s

Near distance of a normal eye: d_0=250 mm

1



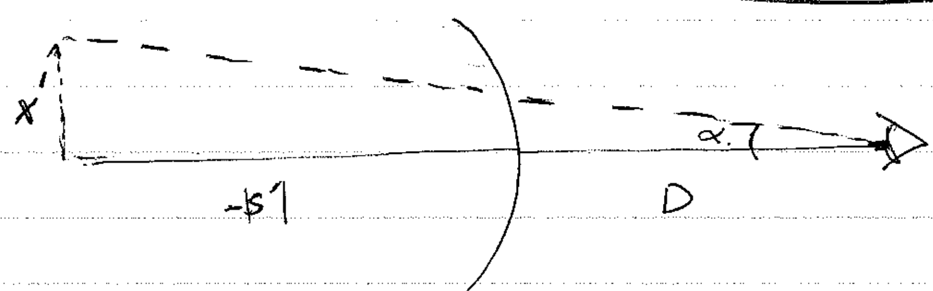
(a) $S = 2|R|$

$$\frac{n}{s} + \frac{1}{s'} = -\frac{1-n}{|R|} \Rightarrow \frac{n}{2|R|} + \frac{1-n}{|R|} = \frac{1}{s'}$$

$$\boxed{s' = \frac{2|R|}{-2+n}}$$

$s' > 0$ fr $n > 2$ (real image)
$s' < 0$ fr $n < 2$ (virtual image)

(b)

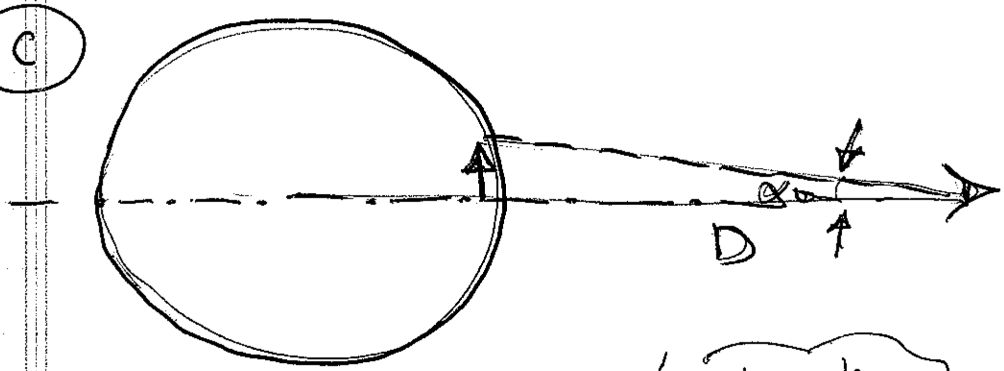


$$\frac{x'}{x} = -\frac{s'}{s} = \frac{1}{2-n}$$

$$\alpha \approx \frac{x'}{-s' + D} = \frac{x'}{D + \frac{2|R|}{2-n}} = \frac{-\frac{s'}{s} h}{D + \frac{2|R|}{2-n}} = \frac{\frac{1}{2-n} h}{D + \frac{2|R|}{2-n}}.$$

$$\boxed{\alpha = \frac{h}{(2-n)D + 2|R|}}$$

(c)



$S = 0$ $\qquad$ $S' = 0$ $\qquad$ $\alpha_0 = \frac{h}{D}.$

$$M_\alpha = \frac{\alpha}{\alpha_0} = \frac{D}{(2-n)D + 2|R|}$$

$$D(1 - M(2-n)) = M\,2R$$

$$D = 2|R| \times \frac{M}{1 - M(2-n)}$$

$M = 1.25$ $\qquad$ $n = 1.3$ $\qquad$ $|R| = 10$ cm

$$D = 2|R| \times 10 = 200 \text{ cm}$$

4.38

Glass (Crown) 510:635 $\Rightarrow$ $n_{d1} = 1.510$ $V_1 = 63.5$

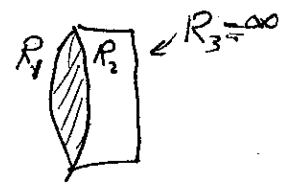
(Flint) 620:364 $\Rightarrow$ $n_{d2} = 1.620$ $V_2 = 36.4$

$f_1 V_1 + f_2 V_2 = 0.$ $\qquad \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f} = \frac{1}{100\text{ cm}}$

$\Rightarrow \frac{1}{f_1} = \frac{-V_1}{V_2 - V_1} \frac{1}{f}$ $\qquad \frac{1}{f_2} = \frac{V_2}{V_2 - V_1} \frac{1}{f}$

$\frac{1}{f_1} = \frac{-63.5}{36.4 - 63.5} \times \frac{1}{f}$ $\qquad \frac{1}{f_2} = \frac{36.4}{36.4 - 63.5} \frac{1}{f}$

$\frac{1}{f_1} = 2.343 \times \frac{1}{f}$ $\qquad \frac{1}{f_2} = -1.343 \times \frac{1}{f}$



$\frac{1}{f_1} = \frac{2.343}{100} = (1.510 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right).$ $\Rightarrow$ $\frac{1}{R_1} - \frac{1}{R_2} = 0.0459$

$\frac{1}{f_2} = -\frac{1.343}{100} = (1.620 - 1)\left(\frac{1}{R_2} - \frac{1}{\infty}\right) =$ $\qquad \frac{1}{R_2} = -0.0216$

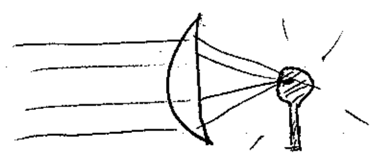
$R_2 = -46.16$ cm $\qquad R_1 = 41.25$ cm.

#3 find f# of each lens.

	1	2	3	4
$\frac{f}{D}$	4	5	4.7	4

- So lenses 1 & 4 have the lowest f#.

- However, for focusing rays from sun ($S \to \infty$). a plano-convex lens works best (minimizes the spherical aberration). So lens 1 is the one to be used as



#4 Read the text & the class notes