# Appendix

## A HP 3585A Spectrum Analyzer

The Hewlett-Packard 3585A is a digital spectrum analyzer with a bandwidth of 20 Hz - 40 MHz. It can be operated manually by the front panel controls, or by computer via the IEEE-488 interface. The manual method is the subject of this appendix.

The entry keys of the front panel are divided into eight groups: INPUT, ENTRY, SWEEP, TRACE, MARKER/CONTINUOUS ENTRY, RBW-VBW-ST, STATUS, and TRIGGER. The pertinent keys in each of these groups will be discussed below. Most keys are designed so that they will be lit, or a corresponding entry on the CRT display highlighted, when on or in effect.

Before discussing the key panel, loo at the CRT display. The horizontal axis is in units of frequency, and either the START and STOP frequencies or the CENTER frequency and frequency SPAN will be given below the bottom grid line. The vertical scale gives the power of the signal in logarithmic units (dBm, dBV, or dB) with the power corresponding to the top grid line given by REF, the values for the other lines decreasing from this value by the amount given in dB/DIV, both displayed in the upper left corner. (Remember that a signal ratio of 10 dB implies an electrical power ratio of 10, while a ratio of 20 dB implies a *voltage* ratio of 10. For most optical detectors, it is the output voltage that is proportional to the optical power in.) The RANGE determines the noise level due to the internal amplifiers. The upper right corner usually gives the MARKER position, but may display other information, and will be discussed later. The meaning of the entries on the bottom line will likewise be discussed below. The CRT is also used to display various messages, which will generally be accompanied by a warning tone. For example, every two minutes or so the instrument will execute a self-calibration routine and "CALIBRATING" will briefly appear on the screen; similarly, if front panel operation is attempted while the device is in the remote mode, the message "HP-IB OPERATION ONLY" will be displayed.

#### The Eight Panel Groups

- 1. INPUT. Proper settings and connections of the INPUT group are required to obtain a trace, and to prevent overloading. For low frequency applications, the IMPEDANCE should be set to 1 MΩ; the 50  $\Omega$  and 75  $\Omega$  inputs need to be used only when investigating frequencies of >10 MHz and when the source is capable of driving such an impedance. The signal source must be connected to the input jack corresponding to the active IMPEDANCE setting. When AUTORANGE is on, the input amplifier RANGE will be adjusted to suit the input signal; with REF LEV TRACK on the power level corresponding to the top of the display grid will follow any adjustments of the RANGE (whether this is due to AUTORANGE, or to any manual STEP KEY or numerical entry).
- 2. ENTRY. This group is the most important of the eight. In particular, the REFERENCE LEVEL, dB/DIV, START FREQ/STOP FREQ and/or CENTER FREQUENCY/FREQUENCY SPAn may be set using the STEP, numerical, and unit keys. To change the value of any of these parameters, press the corresponding key. The corresponding CRT entry will then be highlighted, and the value may be changed either incrementally using the STEP keys (which usually change the parameter by a factor of 2 or in a 1-2-5-10 or 1-3-10 sequence) or by numerical entry from the keypad. If the new value is not allowed a bell will sound and possibly an error message will be displayed; the parameter will remain highlighted



Figure 1: Layout of HP Spectrum Analyzer front panel

awaiting an acceptable entry. A change implemented by the STEP keys will be effected immediately, while a numerical entry will not take effect until an appropriate unit is entered. In either case, the CRT entry will remain highlighted until a units key has been pressed.

- 3. SWEEP. The SWEEP (i.e., the scanning of the internal reference oscillator through the appropriate range of frequencies) may be either CONTinuous (a new sweep beginning as soon as the last one is completed) or SINGLE (sweeping stops when the STOP FREQ is reached). In either case the SWEEPING light will be lit and the CRT trace will be continually updated while the sweep is in progress. The MANUAL ENTRY mode is rarely useful.
- 4. TRACE. The 3585A can store two traces: A, which is continually updated during sweeping, and B, which can be used for permanent storage. A trace must first be obtained in the A trace, then it can be transferred to B by pressing the STORE  $A \rightarrow B$  key. Either or both can be viewed at any time, as can their algebraic difference A B, by activating the appropriate key. The B trace is particularly useful for storing a good trace for later plotting, or for storing baseline noise trace which can then be subtracted from the A signal trace.
- 5. MARKER/CONTINUOUS ENTRY. The MARKER is the highlighted pixel on the spectrum trace which can be positioned by the CONTINUOUS ENTRY knob. The frequency and signal amplitude at its position on the trace will be given in the upper right corner of the CRT when it is active. When the COUNTER feature is active the trace will stop for  $\approx 0.1$  s when the marker frequency is reached and the device will measure the frequency (to an accuracy of 0.1 Hz) of the strongest signal within one resolution bandwidth (RBW

- see below) of the marker frequency. The other features in this group are of little interest for our purposes.

- 6. RBW-VBW-ST. The RESolution Bandwidth is the frequency range allowed through the input filters to the system amplifiers, it is thus the displayed width of any input signal of bandwidth less than the RBW value displayed at the lower left corner of the CRT. It can be reduced to resolve closely spaced signal or increased to speed up the sweep rate if the extra resolution is not needed. The VIDEO BandWidth (VBW bottom center of the CRT) is, qualitatively speaking, the inverse of the time over which a particular element of the display is average; as such, it can be reduced to average fluctuations in the noise level which might mask weak signals, or increased to reduce the SWEEP TIME. With the COUPLES TO SPAN feature active, the SWEEP TIME (ST lower right corner of the display) will be automatically set to the minimum value for which the instrument can still take calibrated measurements; if the ST is too small for the RBW and VBW, the UNCAL LED will light, and the SWEEP TIME increased until it goes off. The RBW and VBW can be varied in a 1-3-10 sequence only; the ST can be set to any multiple of 0.2 s.
- 7. *STATUS*. When the 3585A is being controlled via the IEEE-488 [also know as the HP-IB (Hewlett-Packard Interface Bus) and GPIB (General Purpose Interface Bus)] interface on the rear of the instrument, the REMOTE LED will be lit and the front panel will be disabled. The other three LEDs indicate the status of the instrument with respect to the bus. REMOTE control may be overridden and front panel operation restored by pressing the local key.
- 8. TRIGGER. The FREE RUN mode should be used.

Plotting. Though not an entry key group, the capability to generate a hard copy plot of the displayed frequency spectrum is a useful feature of the 3585A. To obtain a plot, an X-Y recorder must be connected to the X and Y PLOTTER OUTPUT connectors on the back panel of the instrument (if the recorder provides a remote pen lift feature, this should be connected to the Z output - note however that there may be some compatibility problems). The plot is initiated by pressing the RECALL (on) key [lower left corner of the ENTRY block] then the 8 (plot1) key. An analog copy of the trace will be sent to the output jacks at a rate which a typical recorder will be able to follow. Output voltages will range from 0 V (START FREQUENCY) to +10 V (STOP FREQUENCY) for X, and from 0 V (lower grid line) to +10.64 (REFERENCE LEVEL = 10.4 V) for Y. Set the recorder inputs ranges accordingly. The outputs are set to the high limits after each plot, so zero the recorder at the upper right corner of the page.

A simple demonstration - "Watching the radio". To demonstrate the operation of the 3585A, connect a short piece of unshielded cable to the  $1M\Omega$  input. Tune the START and STOP frequencies to locate the AM radio band (0.5 - 1.6 MHz). Position the marker on one of the stations and measure the frequency of the station using the COUNTER. Watch the frequency and amplitude of the signal over several measurements. Can you see why these are 'amplitude modulation' stations rather than 'frequency modulation'? Can you find the FM stations?

The program "HP 3585A Dump A Trace" has been written to drive the HP 3585A and to collect the appropriate data for this experiment. Be sure to start the spectrum analyzer first then the computer. Then follow the instructions on the bottom of the screen.

Suggestions. You will want to take several data points at each position to get an idea of the accuracy of your measurements. Also, you may need to change the CENTER/SPAN or

START/STOP frequencies to keep the signal on the trace. Similarly, the RBW, VBW, and dB/DIV may need to be adjusted to suit the signal. If you have problems with spurious signals, the A-B feature is very useful - see above.

## **B** Scanning Fabry Perot Interferometer

This device uses a Fabry Perot cavity with an adjustable mirror spacing to scan the longitudinal positions of one of the mirrors with a piezo driven by a ramped voltage. By adjusting the mirror separation with the piezo in time, the various transmitted frequencies are being swept. Thus, the transmission versus time as displayed on an oscilloscope actually represents transmission versus frequency (uncalibrated however). Calibration is obtained by making the ramped voltage into a sawtooth waveform. The period of the sawtooth (which can be adjusted) corresponds to a fixed frequency interval of 2 GHz (referred to as the Free Spectral Range of the Scanning Fabry Perot).



Figure 2: Scanning Fabry Perot output. Top: Optical Transmission T vs. frequency f; Bottom: Corresponding Piezo voltage V vs. time t

#### **Adjustable Features**

Vertical gain: Amplitude adjustment of the detector output.

*Dispersion*: Frequency scale adjustment (equivalent to changing the observing oscilloscope's time base)

*Centering*: Adjustment of the "frequency" (or time) position without changing scales. This is achieved by adjusting the DC bias to the piezo.

# C Lock-in Amplifiers

The lock-in amplifier is an important tool because it simultaneously accomplishes amplification and noise reduction. Lock-ins can thus be used to detect AC signals down into the nanovolt regime that would otherwise be obscured by a much greater noise source. Typically, the lockin applies what is called phase sensitive detection in order to single out that portion of the signal that occurs at a certain reference phase and frequency. Noise components that appear at frequencies other than the reference will be rejected. The understanding and and operation of the lock-in will be explained by examining a generalized optical setup in Fig. 3.



Figure 3: Generalized lock-in amplifier as used in an optical setup

In a typical optical setup the reference signal is generated by a "chopper", a motor driven wheel with wide spokes inserted into the beam so as to modulate the intensity like a square wave:

$$E_{ref}(t) = \begin{cases} +1 & \text{for } 0 \le t < \frac{\pi}{\omega_{ref}} \\ -1 & \text{for } \frac{\pi}{\omega} \le t < \frac{2\pi}{\omega_{ref}} \end{cases}$$
(1)

In nonoptical settings, the reference is sometimes created by a function generator. The reference is put through a phase shifter that allows one to vary the relative phase  $\phi_{ref}$  between the reference and signal while the weak signal (given by  $E_{sig}(t) = E_s \cos(\omega t + \phi)$ ) is amplified. One should note that since the optical signal is chopped,  $\omega = \omega_{ref} = 2\pi\nu = 2\pi/T$ . The signal and reference are then compared in a phase detector, which effectively multiplies the main signal with the reference. This product signal is then sent through a low pass filter (with time constant  $\tau = RC >> T$ ) to give the time-averaged output (where  $< > = \frac{1}{T} \int_0^T dt$ ):

$$\langle E_{out} \rangle = \langle E_s \cos(\omega t + \phi - \phi_{ref}) \rangle |_0^{\pi/\omega} - \langle E_s \cos(\omega t + \phi - \phi_{ref}) \rangle |_{\pi/\omega}^{2\pi/\omega}$$
$$= \frac{-2E_s}{\pi} \sin(\phi - \phi_{ref})$$
(2)

When the phases are adjusted properly, the ouput becomes a DC level related to the signal amplitude  $E_s$ . With this in mind, one appropriately inputs the signals into a lock-in. The two phases are placed 90 ° out of phase and adjusted for a minimum output before turning the 90 ° off. This ensures the optimal phase alignment. The signal level is adjusted via the sensitivity knob and the noise reduction is controlled by adjusting the time constant  $\tau$ .

### D Computer-based Data Acquisition

A few of the labs (such as the white light interferometry and the wavemeter labs) involve taking data with a computer. For our purposes, the data acquisition board in the computer is a DASH-16 model offered by Keithley Metrabyte.

The DASH-16 consists of a set of A/D converters (fast AD 774 12 bit converters), and relatively fast D/A drivers. It can be operated with 8 differential or 16 single ends inputs, and can be set in unipolar or bipolar modes (i.e. strictly positive or both positive and negative voltage inputs). The maximum throughput conversions is 100,000/sec.

The A/D conversion may be triggered three ways, by software command (which is generally used), by an on-board programmable timer, or by an external trigger pulse. The transfer of the data can also be accomplished in three ways, by program control, by interrupt service routine, or by direct memory access (DMA). All modes are selectable in software.

The DASH-16(F) has a wide variety of available input ranges which are selected by DIP switch on the board. Selectable ranges include +1v, +2v, +5v, +10v, and +0.5v, +1v, +2.5v, +5v, +10 volts. Other user selectable input ranges can be obtained by simply installing a single gain setting resistor.

The data acquisition software is called DASwiz. When starting it with the desktop icon be sure to enable macros. A new worksheet is necessary to be loaded in before setting the parameters for data acquisition. Then click on the "DAS Wizard" button located on the left, choose "New Task", and "Analog Input". In the "Scan Tab" set the scan channels to 0 and 1, specify how many data points you want to collect. Finally, choose a scanning rate that is able to resolve the beat note you want to record.

## **E** Polarization Matrices

Matrices are used as a tool to compute changes of properties of light as it propagates through optical elements. There are the "ABCD" matrices used to calculate the changes of beam parameters (inclination of the beam with respect to the axis of an optical system, distance of beam from the axis) as light propagates through a system of optical components (lenses, dielectrics, mirrors). The same "ABCD" matrices are used to calculate the changes in Gaussian parameters (beam width w and radius of curvature R).

Matrices can also be used to calculate changes in temporal characteristics of Gaussian beams (pulse duration and chirp parameter) as the light propagates through phase modulators and dispersive media. The  $2 \times 2$  matrices representing the temporal characteristics of the media can be combined with the "ABCD" matrices in a more complex algebra of  $4 \times 4$  matrices to compute the propagation of pulsed beams through optical components.

Another very popular matrix representation is for the propagation of beams through dielectric layers. The transmission and many reflections of the rays can be reduced to simple algebra. Finally, Jones matrices are used to compute the change in state of polarization of light propagating through various optical components.

The common point of nearly all methods is that the light is represented by a column matrix  $\mathcal{L}$ 

$$\mathcal{L} = \begin{bmatrix} x \\ y \end{bmatrix}. \tag{3}$$

The elements x and y can represent the polarizations, the ray vector  $(r, \alpha)$ , the transmitted and reflected fields and so forth, depending on the usage. The particular properties relating to propagation through an element  $\mathcal{M}$  or an element  $\mathcal{N}$  (which can sometimes be propagation through vacuum) are represented by  $2 \times 2$  matrices.

$$\mathcal{M} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}; \ \mathcal{N} = \begin{bmatrix} e & f \\ g & h \end{bmatrix}.$$
(4)

Matrix products are arranged from right to left. The state of light  $\mathcal{L}_1$  after propagation through  $\mathcal{M}$  is

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \mathcal{M} \cdot \mathcal{L}_1 = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix};$$
(5)

the result being, of course:

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} ax_1 + by_1 \\ cx_1 + dy_1 \end{bmatrix}.$$
 (6)

Matrices are meant for linear operations. The result of two consecutive operation (propagation through two successive elements – first element  $\mathcal{M}$ , then element  $\mathcal{N}$ ) can simply be represented by the product  $\mathcal{P}$  of the representative matrices.

$$\mathcal{P} = \mathcal{N} \cdot \mathcal{M} = \begin{bmatrix} ea + fc & eb + fd \\ ga + hc & gb + hd \end{bmatrix}.$$
 (7)

## Study of polarization by Jones Matrices

Matrices offer a convenient way to simulate the state of polarization of light, and its evolution as it propagates through elements. We choose here to summarize a method known as "Jones matrices". In this representation, x and y are the (complex) components of the normalized field vector. Let the light be propagating along the z-axis and neglect the transverse structure of the field. This electric field is represented by its complex amplitude oscillating at the light frequency.

$$E(z,t) = \mathcal{J}\mathcal{E}(z,t)e^{i(\omega t - kz)},$$
(8)

where  $\omega$  is the light frequency, k is the wave vector,  $\mathcal{E}(z,t)$  is the amplitude function of the electric field, and  $\mathcal{J}$  is the Jones matrix representing the state of polarization. For the other axis, we choose a left-handed system with the x-axis along the horizontal direction perpendicular to k, and the y-axis along the vertical direction. The polar angle  $\theta$  is in the xy-plane with the x-axis as the reference, as shown in the sketch of Fig. 4. For linearly polarized light along the



Figure 4: Summary of the conventions chosen.

x-axis (horizontally polarized), let

$$\mathcal{J}_x = \begin{bmatrix} 1\\0 \end{bmatrix},\tag{9}$$

while for "vertically" polarized light,

$$\mathcal{J}_y = \left[ \begin{array}{c} 0\\1 \end{array} \right]. \tag{10}$$

The light will be circularly polarized if the x and y components are equal and there is a phase difference of  $\pi/2$  between the two components. For right circular polarization (RCP), the y component leads by  $\pi/2$ .

$$\mathcal{J} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1\\i \end{bmatrix},\tag{11}$$

or more generally,

$$\mathcal{J} = \frac{1}{\sqrt{2}} \begin{bmatrix} e^{i\phi} \\ e^{i\phi+\pi/2} \end{bmatrix} = \frac{1}{\sqrt{2}} e^{i\phi} \begin{bmatrix} 1 \\ i \end{bmatrix}.$$
 (12)

Intermediate states are elliptically polarized. For a light polarized along x, with some right circular component, the Jones matrix might for instance be

$$\frac{1}{\sqrt{5}} \left[ \begin{array}{c} 2\\ i \end{array} \right]. \tag{13}$$

It is only when the x- and y-axes are the principal axes of the ellipse that the two components are  $\pm \pi/2$  out of phase, as in the previous example. This will not be the case in general.

**Orthogonality** Two states of polarization are said to be orthogonal when their Jones matrices  $\mathcal{J}_1$  and  $\mathcal{J}_2$  are orthogonal in the sense that their inner product is equal to zero.

$$\mathcal{J}_1 \cdot \mathcal{J}_2^* = x_1 x_2^* + y_1 y_2^* = 0 = \mathcal{J}_1^* \cdot \mathcal{J}_2.$$
(14)

It is easily verified that the pairs left and right circular polarization and linear polarization along x and y have this property.

**General definition** Referring to Eq. (8), the field can be expressed by its x and y components.

$$\mathbf{E} = \mathcal{E}_{x}e^{i\phi_{x}}\mathbf{\hat{x}} + \mathcal{E}_{y}e^{i\phi_{y}}\mathbf{\hat{y}} \\
= \frac{1}{\sqrt{\mathcal{E}_{x}^{2} + \mathcal{E}_{y}^{2}}} \begin{bmatrix} \mathcal{E}_{x}e^{i\Delta} \\ \mathcal{E}_{y}e^{-i\Delta} \end{bmatrix} e^{\frac{\phi_{x} + \phi_{y}}{2}}e^{i(\omega t - kz)}$$
(15)

where  $\Delta = (\phi_x - \phi_y)/2$  is half the phase difference between the two field components. The field is linearly polarized when  $2\Delta = \pm 2m\pi$ , where *m* is an integer. Indeed, the field components are then either in phase (even *m*) or 180° out of phase (odd *m*). The orientation of the field vector depends on the relative ratio of the amplitudes  $\mathcal{E}_x$  and  $\mathcal{E}_y$ .

The field is right elliptically polarized when  $2\Delta = (2m+1)\pi/2$ . If the amplitude components are also equal, the field is right circularly polarized. With the observer looking back at the source, the tip of the electric field vector rotates in a clockwise or positive direction with angular frequency  $\omega$ . The photon carries one unit  $\hbar$  of angular momentum.

Table 1: Some examples of Jones	matrices.
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Linear optical element	Jones matrix
Horizontal $(x)$ polarizer	$\left[\begin{array}{rrr}1&0\\0&0\end{array}\right]$
Vertical $(y)$ polarizer	$\left[\begin{array}{cc} 0 & 0 \\ 0 & 1 \end{array}\right]$
Linear polarizer 45°	$\frac{1}{2} \left[ \begin{array}{rr} 1 & 1 \\ 1 & 1 \end{array} \right]$
Half wave plate (fast axis along $y$ -axis)	$\left[\begin{array}{cc} e^{i\frac{\pi}{2}} & 0\\ 0 & e^{-i\frac{\pi}{2}} \end{array}\right]$
Quarter wave plate (fast axis along $y$ -axis)	$\left[\begin{array}{cc} e^{i\frac{\pi}{4}} & 0\\ 0 & e^{-i\frac{\pi}{4}} \end{array}\right]$
Mirror (plane of incidence along $x$ -axis)	$\left[ egin{array}{cc}  ho_\parallel e^{i\phi_\parallel} & 0 \ 0 &  ho_\perp e^{i\phi_\perp} \end{array}  ight]$

**Change in coordinates** As mentioned previously, an elliptical state of polarization is recognized most easily if the axis of the ellipse coincide with the reference axis x and y. A useful transformation is the rotation operator  $\mathcal{R}(\theta)$  which rotates the frame of reference by an angle  $\theta$  about the z-axis.

$$\mathcal{R}(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}.$$
 (16)

One can also transform from the linear basis to the circular basis, via the transformation matrices:

$$\begin{cases} J_{+} \\ J_{-} \end{cases} = \frac{1}{2} \begin{bmatrix} 1 & i \\ 1 & -i \end{bmatrix} \begin{bmatrix} J_{x} \\ J_{y} \end{bmatrix}, \text{ and} \\ \begin{cases} J_{x} \\ J_{y} \end{cases} = \begin{bmatrix} 1 & 1 \\ -i & i \end{bmatrix} \begin{cases} J_{+} \\ J_{-} \end{cases}.$$
(17)

**Transformation matrices** As with most matrix representations, the transformation of the state (of polarization) of the light by an element is represented by a  $2 \times 2$  matrix. Jones matrices for some useful linear optical elements are given in Table 1.

**Example** Let's look at a simple example of light of arbitrary polarization  $\mathcal{L}_i$  followed by a linear polarizer  $\mathcal{P}$  set at an angle  $\theta_p$ , then a half-wave plate  $\mathcal{H}$  with its slow axis at angle  $\theta_w$ . The matrices  $\mathcal{P}$  and  $\mathcal{H}$  start with their respective axes along the x-axis since that is also our

reference for the angles. The final state of polarization  $\mathcal{L}_f$  is

$$\mathcal{L}_f = \mathcal{R}^{-1}(\theta_w) \cdot \mathcal{H} \cdot \mathcal{R}(\theta_w) \cdot \mathcal{R}^{-1}(\theta_p) \cdot \mathcal{P} \cdot \mathcal{R}(\theta_p) \cdot \mathcal{L}_i$$

With these matrices and well-defined conventions, it should be possible to exploit Linear Algebra to keep track of the characteristics of the beam as it evolves/propagates.