Lens Aberrations and Ray Tracing

1 Background

When beam quality is a concern in an optical setup, lens aberrations are an undesired side-effect. Which aberration affects a particular experiment is not always as obvious as one would like. Take for example, the simple goal to concentrate the maximum amount of optical power at a certain point using a lens. The first "obvious" requirement is that the initial beam should be well collimated, sent along the axis of the optical system, and free of spherical aberrations. However, the power may still not be as high as you might expect at the focus. If you are using white light, the various wavelengths will focus at different points along the axis (chromatic aberration). Or, if using short pulses, the central ray will reach the focus later than the marginal ray. You will learn that these two effects are in fact . . . identical.

Where would *spherical aberrations* be an issue? Many times an experiment will require a well-collimated beam or focusing as much energy as possible into a point, situations that mandate minimal spherical aberrations. Which lens should be used? Bi-convex or plano convex — and in the latter case, which orientation? Another case would be the common practice of using spatial filters to improve the optical quality of an illuminating system. Spherical aberrations within the spatial filter assembly can lead to uncorrectable distortion (concentric fringes).

Where would *chromatic aberrations* be an issue? On a general level, certain applications may require color imaging without having multicolored "halos" from chromatic aberration. What are the best methods for focusing all wavelengths by the same amount? A more specific concern with chromatic aberration would be the aforementioned distortion caused by broad bandwidth ultrashort pulses.

The intent of this laboratory is to study certain key aberrations, get a "feeling" for their importance, and find ways to minimize or even eliminate them.

2 Theory

2.1 The Focal Length of a Lens

In paraxial imaging, the object and image distances S and S', respectively, are related to the focal length f of a lens through the thin lens formula.

$$\frac{1}{S} + \frac{1}{S'} = \frac{1}{f}. \tag{1}$$

For $(S + S') = \kappa$ (a constant, $\kappa \geq 4f$ for real images) this leads to the following:

$$\begin{aligned} \frac{1}{S} + \frac{1}{\kappa - S} &= \frac{1}{f} = \frac{\kappa}{S(\kappa - S)} \\ S^2 - S\kappa + f\kappa &= 0. \end{aligned} \tag{2}$$

Eq. (2) has two solutions for S (S_1 and S_2). This means that at a given distance from the object to the image plane, there are two possible lens positions which yield a sharp image. Note that in the paraxial approximation, the focal length of a thin lens is given by the lens maker's formula.

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right), \tag{3}$$

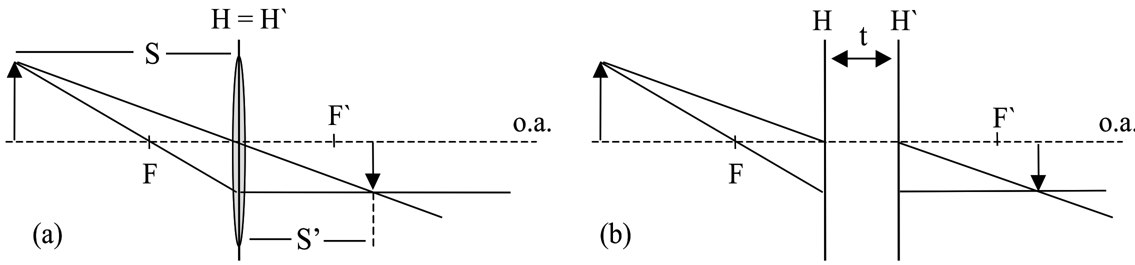

Figure 1: (a) ray tracing through a thin lens and (b) through a system characterized by the principal planes H and H'.

where n is the refractive index of the glass material and r_1, r_2 are the radii of curvature for the two sides of the lens. Due to dispersion (i.e. $n = n(\lambda)$ or $n = n(\omega)$), the focal length of a simple lens varies with wavelength (chromatic aberration).

To find the focal length f, measure κ and ΔL where ΔL is the difference of the two lens positions that produce a sharp image. Calculate the focal length f from κ and ΔL.

2.2 Principal Planes

It is convenient to describe complex imaging systems, for example a thick lens, by means of principal planes. If the two principal planes H and H' and the common focal length f (measured from the principal planes) are known, ray tracing and image construction from a thin lens can be applied, see Fig. 1. Note that for a thin lens the two principal planes coincide and are located in center of the lens. The rays from the object are traced up to H and and continue from H'. The location of the principal planes with respect to the optical components of the system and their separation t can be obtained using optical matrices, see, for example [**?**], Chapter 3.

2.3 Spherical Aberrations

The focal length of a simple, spherical lens varies with the distance h from the optical axis at which the rays enter the lens.

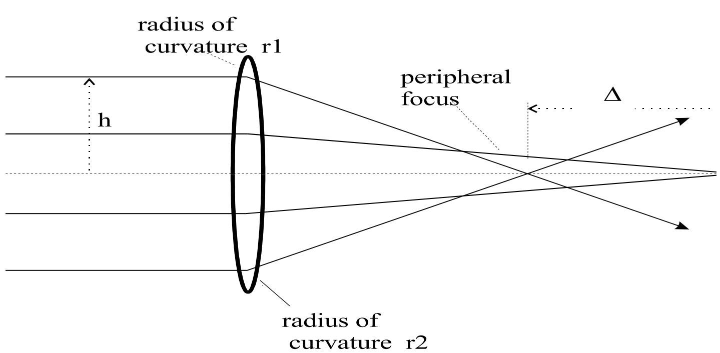

Figure 2: Spherical aberration

The difference Δ between the focal length for paraxial rays, f_0, and rays being incident on the lens a distance h from the optical axis, $f(h)$, is approximately given by:

$$\Delta = [f_0 - f(h)] \approx \frac{1}{2}Kh^2. \tag{4}$$

The parameter K is a characteristic measure of the spherical aberration of the focusing element. For a thin spherical lens it can be written as:

$$K = \frac{1}{4f_0}\frac{1}{n(n-1)}\left[\frac{n+2}{n-1}q^2 + 4(n+1)qp + (3n+2)(n-1)p^2 + \frac{n^3}{n-1}\right], \tag{5}$$

where q is generally called the shape factor and p is the position factor.

$$\begin{aligned} q &= \frac{(r_2+r_1)}{(r_2-r_1)}, \text{ and} \\ p &= \frac{(S'-S)}{(S'+S)}. \end{aligned}$$

Note that these expressions are not symmetrical with respect to the lens orientation.

An alternative way to characterize spherical aberration makes use of the wave optical description of a lens. The phase function of a lens is

$$\Phi(r) = \frac{kr^2}{2f_0} + Ar^4. \tag{6}$$

The first term is the phase term of an aberration-free thin lens and A is a parameter describing spherical aberration.

2.4 Coma aberration

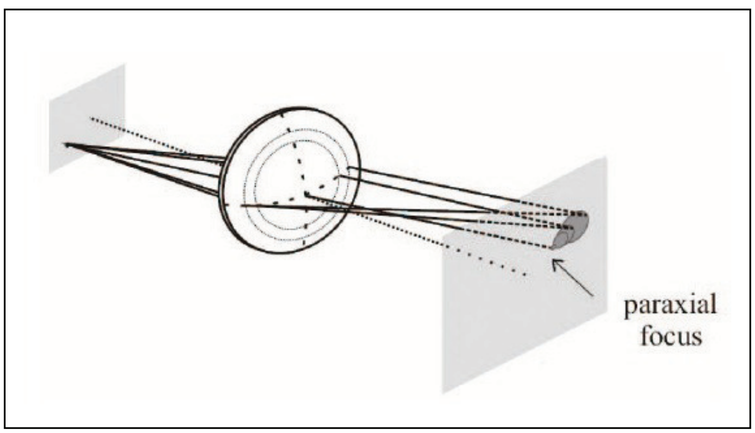

Figure 3: Illustration of Coma Aberration

Coma aberration is a third-order aberration when the light ray is tilted to the principal optical axis of a lens. As a result, an off-axis point source will be imaged as a patch of light instead of a luminous point. The further off-axis, the worse this effect is. The image often exhibits a resemblance to a comet having its tail directed towards or away from the axis, which

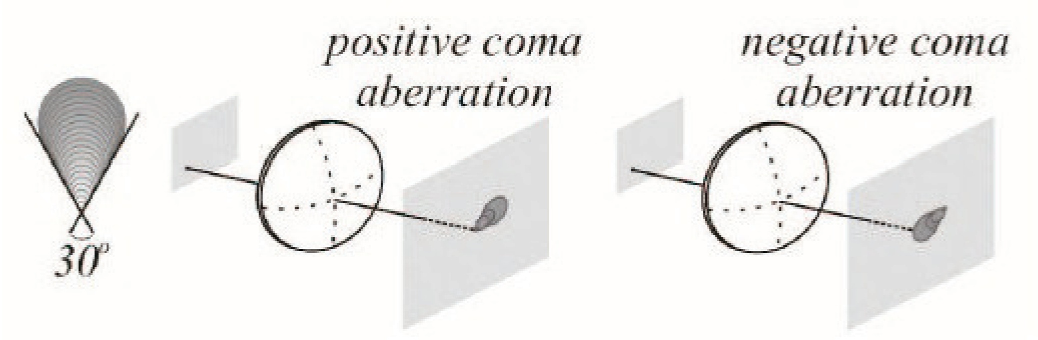

Figure 4: Positive and Negative Aberrations

corresponding to positive and negative coma aberrations (see Fig. 4). From this appearance it takes its name. In comparison with spherical aberration, the coma term can be expressed as:

$$\Delta\Phi(r) \propto r^3. \tag{7}$$

In other words, coma aberration destroys the beam's symmetry around the chief ray and lower the quality of imaging. It can be minimized (and in some cases eliminated) by choosing the curvature of the lens surfaces to match the application. However recently people found out that coma aberration can be used in the generation of Airy beams. More detailed discussion can be found in section 5.3.5.

3 Experimental Setup

3.1 Determining the Focal Length of a Lens

Place a light source (i.e. light bulb), a lens, and a screen on an optical rail. Keeping the bulb and screen fixed is equivalent to fixing $(S + S') = \kappa =$ (a constant), which was said to give a focused image (of the bulb's filament) for the lens placed at two positions along the rail. Using such a setup, one can determine the mean focal length of a given lens. Insert a red transmission filter in front of the light source to minimize chromatic aberrations in this part of the experiment.

Conduct this experiment for the two possible orientations of the plano-convex lens.

To find the difference in the focal length obtained for the two lens orientations, determine the location and separation of the principal planes of your lens. Assume that the refractive index of the lens material $n \approx 1.5$. The thickness of the lens (center) is $d = 20.5$ mm. From the location of the principal planes, explain your measured focal lengths. Define a focal length and a single reference plane (that is, use the thin lens approximation) to be used in the following parts of this lab.

3.2 Chromatic Aberration

Following the same procedure outlined in Section 3.1, you can measure the focal length of a lens for different wavelengths to produce a plot of $f(\lambda)$. To obtain light of different wavelengths, use a set of interference filters in the visible range (~ 400 nm to ~ 700 nm). Each filter should be placed close to the light source and orthogonal to the optic axis. The orthogonality is important since the transmission of an interference filter has a strong angular dependence. (You can easily verify this point by holding a filter at various angles to the overhead light.) Estimate the quantity $\frac{dn}{d\lambda}$ for $\lambda = 500$ nm from the curve $f(\lambda)$.

3.3 Spherical Aberration

To measure the spherical aberration, i.e. to determine K of Eq. (5), we can use the experimental configuration in Fig. 5. Before starting the actual measurement, it is necessary to align the setup.

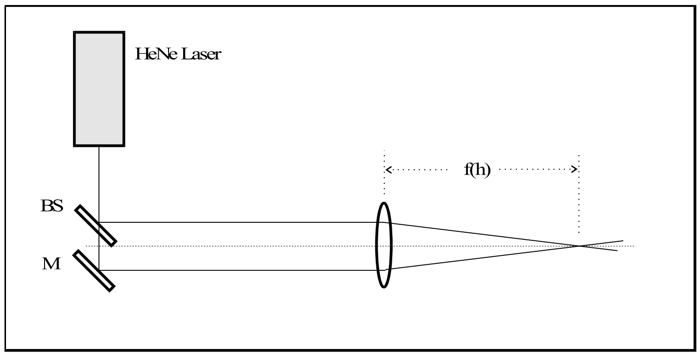

Figure 5: Spherical aberration experimental layout.

Spend some time ensuring parallelism of the two beams. One of the beams should propagate along the optical axis defined by the lens. Remember that one definition of the parallelism of two lines is that they cross at infinity. A few centimeters from the source is *not* a good approximation of infinity. By changing the positions of M, one can vary h. To minimize the error in the "zero h", leave the position of the axis of symmetry (say $h = h_0$) as an adjustable parameter in analyzing the data. The focal length $f(h)$ is measured by translating the lens until the two beams intersect at the observation screen. Change the distance h and measure the various $f(h)$.

For the two possible orientations of the plano-convex lens, measure the focal length as a function of h and plot the results. For each of the two plots of $f(h-h_0)$ determine the aberration constant K from a fit of this data. The fit will give $f_{paraxial}$ at the curve's extremum $h = h_0$. Check if this value agrees with your earlier measurements of the focal lengths. When determining K, be sure you have the right sign (remember that $\Delta = f_{paraxial} - f(h)$). Using the expression for K, determine the index of refraction n. What is your conclusion about how to use the plano-convex lens for focusing a parallel input beam? Give a physical reason. Would the same conclusion be true if you were to image a point at a distance a to a point at distance b?

3.4 Ray Tracing

Develope your own program of ray tracing and compare with your experimental results of spherical aberration. You are recommended to use ABCD matrix method for ray tracing. Please pay attention to the order of the matrices you apply to the initial vector. Some of the matrices you may want to use are given below:

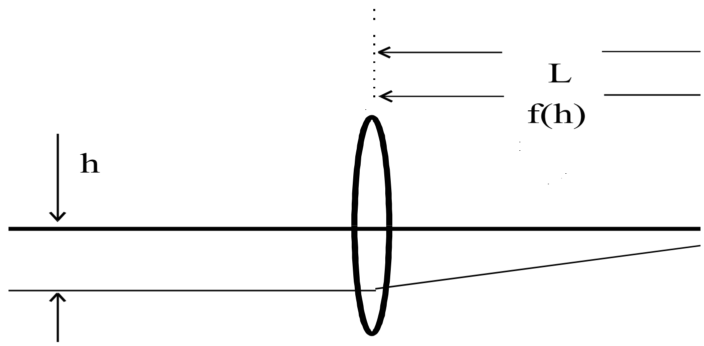

Figure 6: Ray tracing information.

For a ray defined as

$$\begin{pmatrix} y \\ \theta \end{pmatrix},$$

the matrix for propagating a certain distance d in free space is

$$\begin{pmatrix} cc1 & d \\ 0 & 1 \end{pmatrix} \tag{8}$$

; and the matrix for refraction at a curved interface is

$$\begin{pmatrix} 1 & 0 \\ \frac{n_1 - n_2}{R \cdot n_2} & \frac{n_1}{n_2} \end{pmatrix}, \tag{9}$$

where R is the radius of curvature, positive for convex, and n_1, n_2 are the initial and final refractive indices, respectively.

Now it is clear as the input parameters to the ABCD matrices, you will need the radius of curvature R and refractive index n of the lens and so on. Use your imagination and creativity to determine R (without touching the lens surface).

It is sufficient to measure five vectors (h,x) at a constant screen position L for the two lens orientations respectively. Produce a plot of your measurement and the results from the exact ray tracing. Discuss the discrepancies.

Another use of the ray tracing program is to solve the following problem. Find a combination of two lenses that are in contact (lens doublet). This doublet should have the same refractive index and approximately the same focal length as your lens. The should be able to focus a parallel input beam onto a screen with minimum spherical aberration. Measure the "focal length" from the vertex facing the screen. For the search, make use of the optimization features of the ray tracing program. Generate a plot documenting your results. Compare the K value of the doublet with those obtained from your measurements with the single lens.

3.5 Coma aberration and Airy Beams

As shown in Fig. 7, optical Airy beam is a non-diffracting optical beam with an envelope defined by Airy function. The most interesting thing about Airy beam is that its pulse intensity lobes shifts transversely in a quadratic fashion during propagation, while the center of pulse energy remains unshifted.

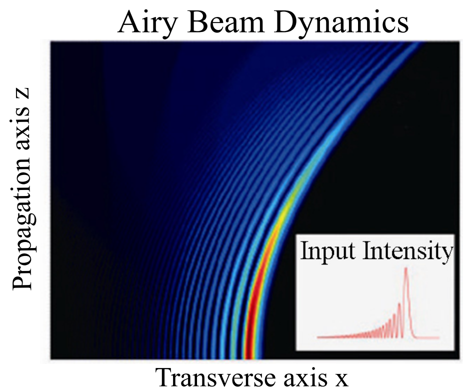

Figure 7: False-colored calibrated intensity profile of the propagation dynamics for an Airy beam

Airy beam in theory can be generated by imprinting a cubic phase modulation on a Gaussian beam and then spatially transforming it using a converging lens. This relies on the fact that the angular Fourier spectrum of an exponentially truncated Airy function is a Gaussian beam modulated with a cubic phase. So far the generation of Airy beams has been demonstrated experimentally using liquid crystal spatial light modulators, specially designed cubic phase masks, three-wave mixing in a asymmetic nonlinear photonic crystals, and optical aberration when a simple lens is tilted. Here you are going to generate the Airy beam using the last method.

Since coma aberration is the only third-order wavefront aberration, our experimental design requirement is to eliminate all aberrations except for the coma. In order to achieve this, you will use a tilted cylindrical beam expander design shown in Fig. 8. In the paraxial approximation,

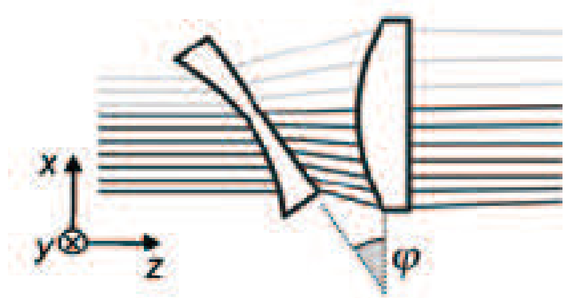

Figure 8: Tilted cylindrical telescope system for the generation of 1D cubic phase modulation.

the collimated input beam remains collimated after exiting the beam expander since the back and front focal points of the lenses coincide. As shown in Fig. 4, you tilt the first diverging cylindrical lens by an angle φ to induce the necessary aberrations. In order to compensate all the undesired aberrations, except the coma, you need to displace the second converging lens in both the longitudial and transverse direction. Provided a pair of 25mm-wide cylindrical lenses of $\pm$ 50mm focal length, for $\varphi = 35^o$ you will displace the second converging lens by 14 mm longitudinally and 1 mm transversly. Subsequently, the beam is spatially Fourier transformed to generated the Airy beam using converging cylindrical lens of focal length f = 100mm.

The generated Airy beam is imaged using a CCD camara. Capture the images on the Fourier plane of the f = 100mm Fourier transforming lens and identify the intensity pattern of Airy beam. Translate your image plane around the focus for a few centimeters(about 4cm), and observe the transverse shift of the highest-intensity lobe as we mentioned above. Try to find out the FWHMs of the Airy highest-intensity lobe as a function of longitudinal distance from the focus. Then compare them with the FWHMs of peak intensity from an propagating Gaussian beam based on your calculations(assume both Airy and Gaussian beams have the same waist and peak intensity at the focus). From here you will get an idea of the nondiffrating feature of Airy beams.

3.6 Michelson Interferometer (optional)

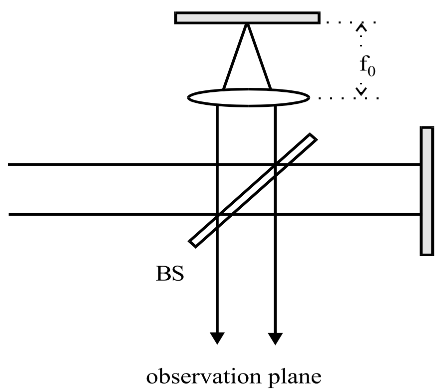

Figure 9: Spherical aberration studies with a Michelson interferometer.

Aberrations cannot be measured for smaller diameter lenses via the method used thus far. Diffraction will interfere with attempts to characterize smaller lenses if the beam diameter is comparable to the size of the lens. The beam used in the following experiment, however, has to completely fill the aperture of the lens under investigation. A Michelson interferometer is used to study the wavefront distortion caused by a lens inserted in one of the arms, see Fig. 9.

With proper alignment, the output beam of the Michelson will be a beam of uniform cross section (provided that the mirrors used for the instrument have a flatness of $\lambda/10$ or better). Proper alignment for this experiment means that the reflected beam should be returning into the He-Ne laser source. With an "ideal" lens in one arm at a (paraxial) focal length away from the mirror, a collimated input should be re-collimated at the output. The lens has no net effect (other than spatial inversion of the beam) so that the uniform wavefront and corresponding interference pattern should not change from the interferometer's "no lens" pattern.

Realistically, the lens aberrations should distort the wavefront and give a unique fringe pattern if interfered with a plane reference wave. Imagine for example that the marginal rays coming back off the mirror are collimated (i.e. for a roughly collimated input this means that the mirror is at the marginal focus f_{marg}). The more paraxial rays would then have been focused behind the mirror, making them within f_{marg} of the lens for "re-collimation" purposes. This means that each ray at a given radial position will come out of the lens at a different angle based on how much closer to the lens the given ray originated. (See Fig. 10). The fringe pattern needs to be analyzed near to the beam splitter (and near to the lens). Why? Repeat this experiment for two different lens orientations.

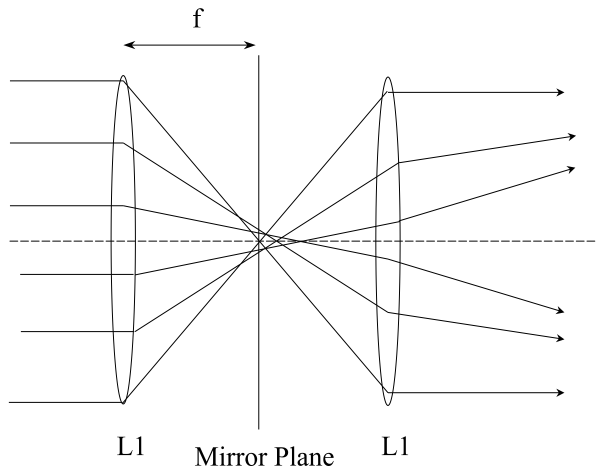

Figure 10: Spherical aberrations within the Michelson Interferometer.

From a quantitative analysis of the fringe pattern one can determine the aberration parameter A. To this end, first measure the radius of the interference rings. A comparison to the fringe spacing from a theoretical analysis provides a value for A. An exact wave-optical treatment of the coherent imaging problem is rather involved and beyond the scope of this lab. We will therefore estimate A.

1. Simple approach: Let us assume that the phase term of the wave after it has passed the second time through the lens is approximately given by $\exp(i2Ar^4)$. Make some qualitative arguments why this makes sense. From this wavefront you can predict the position of the fringes as a function of r and determine A from a comparison to your data.

2. For the theoretically inclined student: For one dimension, derive an expression that describes the wave immediately after the second transmission through the lens. You can use the theory of coherent image formation (see [**?**], Chapter 7). Solve the integral numerically. From a best fit to your data, determine A. If you assume that the wavefront varies parabolically rather than as a r^4 you can solve the integral analytically. Compare this result to the assumption made in 1.

It is interesting to study the reflection from the arm with lens, without interference from the other arm. In the far field, rings are observed. This geometry is similar to that of a telescope or a spatial filter, where fringes can also be observed in the far field pattern, if the lenses has spherical aberration. A simple (somewhat oversimplified) explanation is that the paraxial beams interfere with the marginal ones (cf. Fig. 9 and Fig. 10).

3.7 Michelson Interferometer With A Broadband Source (optional)

Would the above result be different if, instead of a He-Ne laser, you had used a femtosecond Ti:sapphire laser? The output of a 100 fs laser is contained in a pancake shaped volume, of the diameter of the beam, and a thickness of $c \times 100$ fs $= 30\mu$m where c is the speed of light. The pancake is traveling at the speed of light in the direction of its normal. There can only be interference fringes observed where the pancakes merge. The pancake issued from the reference arm (the one without a lens) is undistorted. Because of the thickness of the lens, the beam has been slowed down on axis (the group velocity of light is less in glass than in air) as compared to the edges of the pancake. For instance, if the lens has a thickness of $\ell = 2$ mm, the center of the pancake has been pulled back by $(n-1)\ell \approx 1$ mm, which is much larger than the pulse length (pancake thickness). You conclude thus that, in order to see interference, you will have to adjust the relative position of the pancakes along their direction of propagation, i.e. adjust the delay of the reference arm.

This result applies as well to white light as to radiation from a fs laser, although the interpretation and actual experiment seem easier with a short pulse laser. Discuss which type of aberration this temporal stretch of the focus is related to.

4 Summary

1. Focal Length: Measure the focal length f of the lens to be studied using the white light source and a red filter. Discuss your results in terms of a thick lens and principal planes. Use the thin-lens approximation to define a focal length and a reference plane. Do this for both orientations of the lens. If you measure the radius of curvature for this plano-convex lens, what is the index of refraction n for the lens (based on the lens maker's equation)?

2. Chromatic Aberration: Measure the focal length as a function of wavelength using the interference filters available. Translate this into a plot of n vs. λ to extract $dn/d\lambda$.

3. Spherical Aberration: Using the "h" method for both orientations of the lens to determine spherical aberration. From f vs. h determine K for both cases. Derive a relation between the aberration parameters A and K. Which orientation has less spherical aberration for this situation (where a collimated beam is being focused)? From your K values, determine n.

4. Ray Tracing: Using your own ray tracing program, simulate the experimental setup and compare your results with your measurements of the spherical aberration. Also simulate a lens doublet that will minimize the aberration (i.e. focus the various rays to a single point). Discuss your findings.

5. Michelson Interferometer: Observe and describe the fringe pattern for the "no lens" case and the "normal lens" case. Estimate the value of the aberration parameter K and A for the two different lens orientations.

References

[1] Miles V. Klein and Thomas E. Furtak. *Optics*. John Wiley and Sons, ISBN 0-471-87297-0, New York, 1986.