Problem 1. Nonlinear optical measurements show that an optical glass (SiO$_2$) has $\tilde{\eta}_2 = 1.3 \times 10^{-13}$ esu at $\lambda = 850$ nm. The linear refractive index $n_0 = 1.5$

(a) What is $n_2$ in $\text{cm}^2/\text{W}$ and $\text{m}^2/\text{W}$? (see Appendix C for unit conversions).

(b) What is $\chi^{(3)}$ in SI units?

(c) Estimate the peak index change ($\Delta n$) induced by a modelocked laser operating at 500 mW (average power), 20 fs laser pulse width and 100 MHz repetition rate. The laser (Gaussian profile) is focused to a spot size of $w_0 = 10 \mu\text{m}$.

Problem 2. Extreme nonlinear optics occurs when the incident optical field approaches the characteristic atomic field $E_{at} = e/(4\pi\varepsilon_0)\alpha_0^2$ where $\alpha_0$ is the Bohr radius (read section 1.1 in Boyd). In this regime, we can no longer describe the nonlinearity by nonlinear susceptibilities as the process becomes non-perturbative. At such high electric fields, the atom simply ionizes.

Calculate $E_{at}$ and its corresponding irradiance $I_{at}$. What is the required pulse energy to achieve this irradiance for a 30 fs laser pulse focused to 20 $\mu$m spot size?

Problem 3. Pockel’s Effect: A 2$^{\text{nd}}$ order nonlinear crystal with a known $\chi^{(2)}$, refractive index $n_0$ and a thickness $L$ is used as an electro-optic modulator as shown below. Here a DC voltage ($V_{dc}$) is applied across two transverse electrodes (separated by $d$). Ignoring anisotropy and tensor properties, show that the phase of the transmitted electric field will be modulated according to:

$\Delta \phi(V) = \kappa V_{dc}$

(a) What is $\kappa$ (use SI notation)?

(b) For $\chi^{(2)} \approx 1$ pm/V, find the required $V_{dc}$ to achieve $\Delta \phi = \pi$ for $L = 1$ cm, $\lambda = 500$ nm. Assume $d = 10$ mm, $n_0 = 1.5$. 

$$E = A_0 e^{ikz - i\omega t + i\phi_0}$$
Problem 4. **EFISH: Electric-Field Induced Second Harmonic**

Consider a centrosymmetric and isotropic material (e.g. glass) for which $\chi^{(3)}(\omega_4; \omega_3, \omega_2, \omega_1)$ is known. In an experimental arrangement (as shown in the Figure) this material is sandwiched between two parallel electrodes while an intense laser beam is propagating parallel to the electrodes.

(a) By applying a large d.c. voltage ($V$), some second harmonic generation ($2\omega$) is observed. Explain how this is possible. Note (show that) this corresponds to $\chi^{(3)}(2\omega; \omega, \omega, 0)$

(b) Assuming $\chi^{(3)} \approx 10^{-22} \text{ m}^2/\text{V}^2$, estimate the required voltage to produce a $\chi^{(2)}_{\text{eff}}$ equal to that of KDP ($\chi^{(2)} \approx 1 \text{ pm/V}$). The electrode spacing $d=10$ mm.

(c) In the small signal regime (i.e. when the incident light intensity is very low), show that the phase of the transmitted beam is modulated by the applied voltage. Explain.
Q1. SiO2. $\tilde{n}_2 = 1.3 \times 10^{-13}$ esu @ $\lambda = 850$nm $n_0 = 1.5$

ESU $\rightarrow$ SI.  

$$n_2 (m^2/W) = \frac{4\pi n}{c} \frac{\tilde{n}_2 (\text{esu})}{n} \approx 4.19 \times 10^{-7} \frac{1.3 \times 10^{-13}}{1.5}$$

$$\approx 3.63 \times 10^{-20} \text{ m}^2/\text{W}$$

$$n_2 \text{ (cm}^2/\text{w}) = 1 \times 10^4 \times n_2 \text{ (m}^2/\text{w}) = 3.63 \times 10^{-16} \text{ cm}^2/\text{w}$$

(b) 

$$n_2 \left( \frac{m^2}{W} \right) = \frac{2.83}{n_0^2} \chi_{31} (\frac{m^2}{V^2})$$

from textbook P210 (4.1.20)

$$\Rightarrow \chi_{31} (\frac{m^2}{V^2}) = \frac{n_2 \left( \frac{m^2}{W} \right) \cdot n_0^2}{2.83} = 2.89 \times 10^{-22} \text{ m}^2/\text{V}^2$$

(C) 

$$n = n_0 + n_2 I$$

$$\Delta n = n_2 I.$$

Assume here $w_0$ is for radius not diameter.

$$I_{\text{peak}} = \frac{P_{\text{peak}}}{A}$$

$$A = \pi r^2 = \pi \times (10 \times 10^{-6})^2 \approx 3.14 \times 10^{-10} \text{ m}^2.$$ 

$$P_{\text{peak}} \approx \frac{\text{Powerage} \times 15}{2065 \times 100 \times 106} = \frac{500 \times 10^{-3} \text{ W} \times 15}{20 \times 10^{-15} \text{ s} \times 100 \times 10^6} = 2.5 \times 10^5 \text{ W}.$$ 

Do an approximation for pulse shape.

$$I = \frac{2.5 \times 10^5 \text{ W}}{3.14 \times 10^{-10} \text{ m}^2} \approx 7.96 \times 10^{14} \text{ W/m}^2.$$

$$\Delta n^2 = n_2 I = 3.63 \times 10^{-20} \text{ m}^2/\text{W} \times 7.96 \times 10^{14} \text{ W/m}^2 \approx 2.89 \times 10^{-6}$$
Q2.

\[ E_{at} = \frac{e}{(4\pi \epsilon_0) a_0^2} = \frac{1.6 \times 10^{-19}}{4 \pi \times 8.85 \times 10^{-12} \times (6.29 \times 10^{-11})^2} \approx 5.19 \times 10^{11} \text{V/m} \]

\[ I_{at} = \frac{1}{2} \epsilon_0 c \ E_{at}^2 = 3.5 \times 10^{20} \text{W/m}^2 \]

Do an approximation of pulse Shape:

Pulse Energy \approx I_{at} \times 3 \text{ ofs} \times A = 3.5 \times 10^{20} \text{W/m}^2 \times 3 \times 10^{-15} \text{ s} \times \pi \times (6 \times 10^{-6})^2 \approx 1.32 \times 10^{-2} \text{ J} = 13.2 \text{ mJ} \]

Q3.

(1) \[ \phi(V) = k \ V_{dc} \]

\[ R = \frac{\phi(V)}{V_{dc}} \]

\[ \phi = kL = \frac{2\pi \omega n}{\lambda} \cdot L \]  

Need to calculate \( \Delta n \). ( refractive index change for different w)  

\[ n = \sqrt{\frac{E}{E_x}} = \sqrt{1 + X_{eff}} \]

\[ E_x = \frac{E_0}{X_{eff}} \]

\[ \theta \]

Pulse Energy \approx I_{at} \times 3 \text{ ofs} \times A = 3.5 \times 10^{20} \text{W/m}^2 \times 3 \times 10^{-15} \text{ s} \times \pi \times (6 \times 10^{-6})^2 \approx 1.32 \times 10^{-2} \text{ J} = 13.2 \text{ mJ} \]

(5) \[ V_{dc} = \frac{\phi \ n_0 \ d}{2\pi \ X_{eff} \ L} = \frac{\pi \times 1.5 \times 10^{-3} \text{ m} \times 500 \times 10^{-9} \text{ m}}{2\pi \times 1 \times 10^{-12} \text{ m/V} \times 1 \times 10^{-2} \text{ m}} = 3.75 \times 10^{5} \text{ V} \]
@ for centrosymmetric material, there is no $X^{21}$ effect, however, if we introduce a large DC voltage to the material, the property of the material changes, say, we break the symmetry of the material, and it is possible that we have a SHG, and this process relates to $X^{31}(\omega; w, w, 0)$. $P(\omega) = 3 E_0 X^{31}(\omega; w, w, 0) E(\omega) E(0) = E_0 X^{11 \text{eff}} E^2(\omega)$

$$P(\omega) = 3 X^{31}(\omega; w, w, 0) E^2(\omega) \frac{V_{dc}}{d} = X^{11 \text{eff}} E^2(\omega).$$

$$X^{11 \text{eff}} = \frac{3 X^{31} V_{dc}}{d} = 1 \times 10^{-12} \text{m/V} \Rightarrow V_{dc} = \frac{1 \times 10^{-12} \text{m/V} \times 10 \times 10^{-3} \text{m}}{3 \times 10^{-22} \text{m}^2/\text{V}^2} \approx 3.33 \times 10^7 \text{V}$$

$$P(\omega) = E_0 X^{11}(\omega) + 3 E_0 X^{31}(\omega; w, w, 0) E(\omega) \left( \frac{V_{dc}}{d} \right)^2$$

$$= E_0 \left[ X^{11} + 3 \frac{X^{31}(\omega)}{d} \right] E(\omega) = E_0 X^{11 \text{eff}} E(\omega).$$

$$n_0^2 \sqrt{1 + \frac{X^{11 \text{eff}}}{X^{31}}} = \sqrt{1 + X^{11} + 3 \frac{X^{31} V_{dc}}{d^2}} = \phi \ n_0 \sqrt{1 + \frac{3 X^{31} V_{dc}}{n_0^2 d^2}}$$

$$\approx n_0 + \frac{3 X^{31} V_{dc}}{2 n_0 d^2} \Delta n = \frac{3 X^{31} V_{dc}}{2 n_0 d^2}$$

Phase change: $\Delta \phi = \frac{2 \pi \Delta n}{\lambda} L = \frac{3 \pi X^{31} V_{dc} L}{n_0 \lambda d^2}$

This phase modulation is similar to Pockels' effect, but it is a $X^{31}$ effect not $X^{21}$ effect. When the incident field is small, the $E(\omega)$ field introduced by DC voltage are not ignorable, we should consider $X^{31}(\omega; w, w, 0)$ for $P(\omega)$, as a result, $n$ is modulated, thus the phase of transmitted beam is modulated.