NONLINEAR OPTICS (PHYS 568)

Spring 2022 - Instructor: M. Sheik-Bahae

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Homework #6, Due Monday, April 4

1. Thermal n_2^{eff}

(a) Calculate the nonlocal n_2^{eff} (defined as $\langle\Delta n\rangle/I_0$) due to laser heating of a liquid characterized by its absorption coefficient α (cm^{-1}), density ρ ($gr./cm^3$), heat capacity C_v(J/K/gr.) and thermo-optic coefficient dn/dT (K^{-1}). The laser intensity is $I(t)=I_0 f(t/t_p)$ where I_0 is the peak intensity and $f(t/t_p)$ denotes the normalized temporal profile of the pulse. Thermal diffusion can be ignored in this problem if we assume that the diffusion time is much longer than t_p while being much shorter than the inter-pulse spacing. The latter requirement is for avoiding heat accumulation from pulse to pulse.

(b) Evaluate n_2^{eff} for liquid CS_2 and a pulsed CO_2 laser (λ=10.6 μm) having a square temporal profile (t_p=100 ns). The CS_2 parameters are $\alpha=0.2$ cm^{-1}, ρC_v=1.3 J/K/cm^3, and dn/dT=$-8\text{x}10^{-4}$ K^{-1}.

(c) If the sound velocity (v_s) in CS_2 is 1.5x10^5 cm/sec., what is the largest laser spot-size (w_0) for which the n_2^{eff} obtained in (b) is valid? What happens as the spot size becomes larger than this value?

2. n_2^{eff} due to photo-generation of charge-carriers in semiconductors:

(a) Calculate the n_2^{eff} due to resonant interband charge-carrier generation in semiconductors. The known parameters for the semiconductor are: the band-gap energy E_g, the electron effective mass m^* (for both conduction and valence bands), the valence-to-conduction band absorption coefficient α, and the carrier recombination time τ. This requires a calculation of the electronic density change ΔN (in both bands) due to linear absorption followed by the calculation of the resultant index change Δn from both bands using a harmonic classical electron oscillator (CEO) model. In this simple approach, the electrons in the valence band are considered bound with a resonant frequency $\omega_0=\omega_g=E_g/\hbar$, while the conduction electrons are considered free (ω_0=0). Ignore damping in the CEO models. The governing equation for ΔN is:

$$\frac{d\Delta N}{dt} = \frac{\alpha I(t)}{\hbar\omega} - \frac{\Delta N}{\tau}$$

where $\hbar\omega$ is the incident photon energy and $I(t)=I_0 f(t/t_p)$ is the instantaneous laser intensity. Consider two extreme cases of $t_p >> \tau$ and $t_p << \tau$. (You may assume a rectangular pulse).

(b) Evaluate n_2^{eff} (cm/W) and the effective $\chi^{(3)}$ (esu or m^2/V^2) for GaAs with E_g=1.4 eV, $m^*=0.1\ m_0$, α=100 cm^{-1}, τ=1 ns. The laser wavelength is λ=900 nm and t_p=10 ps.

(c) Do you expect an (nonlinear) absorptive component $\Delta\alpha$ associated with the above index change Δn? Explain (briefly).

Problem 3. Consider an *isotropic* nonlinear material (e.g. silica glass). We are concerned here with the self-phase modulation case where $\chi^{(3)}_{ijkl}(\omega;\omega,\omega,-\omega)$ is involved.

(a) Show that $\chi^{(3)}_{xxxx}=\chi^{(3)}_{xxyy}+\chi^{(3)}_{xyyx}+\chi^{(3)}_{xyxy}$

(b) Let the incident field be represented by $\vec{E}=E_0 e^{-i\omega t}(\hat{x}+e^{i\phi}\hat{y})/\sqrt{2}$ where ϕ is the phase difference between *x* and *y* components. Note that $\phi=m\pi$ and $\phi=(m+1/2)\pi$ describe the linear and circular polarization receptively. Let us now define an effective nonlinear susceptibility through:

$$\vec{P}^{(3)}(\omega)=3\varepsilon_0\chi^{(3)}_{eff}\,|E_0|^2\,\vec{E}$$

(i) Find $\chi^{(3)}_{eff}$ as a function of the $\chi^{(3)}$ tensor elements and the phase angle ϕ. Note, this corresponds to conditions where P is along E. Can we define such a χ_{eff} for all values of ϕ (i.e. elliptical polarization)? *Read Sec. 4.2 (Boyd, 3rd. ed.). You may start with Eq. 4.2.7*

(ii) Find the circular/linear dichroism defined as $\eta=\chi^{(3)}_{eff}(\phi=\pi/2)/\chi^{(3)}_{eff}(\phi=0)$