NONLINEAR OPTICS (PHYC/ECE 568)

Spring 2022 - Instructor: M. Sheik-Bahae
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Homework #4, Due Monday, March 21

Problem 1. *SHG in KDP:*
a. Calculate the type-I phase matching angle for SHG in KDP using 1.06 μm output of a Nd:YAG laser.

b. For a beam radius w_0=500 μm, calculate the aperture length defined as $l_a=\sqrt{\pi}\, w_0/\rho$ where ρ is the Poynting vector walk-off angle. Obtain the aperture length for w_0=15 μm and discuss the role of additional limitations that may be imposed due to diffraction of the beam.

Problem 2. SHG Bandwidth:
a. Calculate the bandwidth $\Delta\omega$ associated with a phase-matched SHG process in terms of the group velocities $v_g(\omega_1)$ and $v_g(2\omega_1)$. In the low-depletion approximation, this corresponds to the width of the $Sinc^2$ function which is taken to be $\delta(\Delta kL)=2\pi$ with L denoting the length of the nonlinear crystal.
Hint: Use the first-order term in the Taylor series expansion of $\Delta k(\omega)$.

b. Discuss how your results in (a) explains the limitation on the SHG-efficiency when ultrashort laser pulses are used.

Problem 3. What about the fundamental wave?
Consider the case of a phase-matchable SHG process; but instead of being concerned about the second-harmonic beam (at 2ω), we would like to determine the fate of the transmitted fundamental field at ω (see also problem 2.20 in Boyd, 3rd ed.).
(a) Start with the coupled amplitude equations (i.e. Eqns. 2.7.10-11 in Boyd). Eliminate A_2 to obtain the following nonlinear differential equation for A_1:

$$\frac{d^2A_1}{d\,z^2}+i\Delta k\frac{dA_1}{dz}+\Gamma^2A_1\left[2\,|A_1/A_1(0)|^2-1\right]=0$$

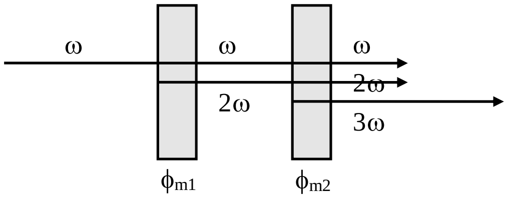
where $\Gamma^2=4d^2\omega_1^2|A_1(0)|^2/(c^2n_1n_2)$.

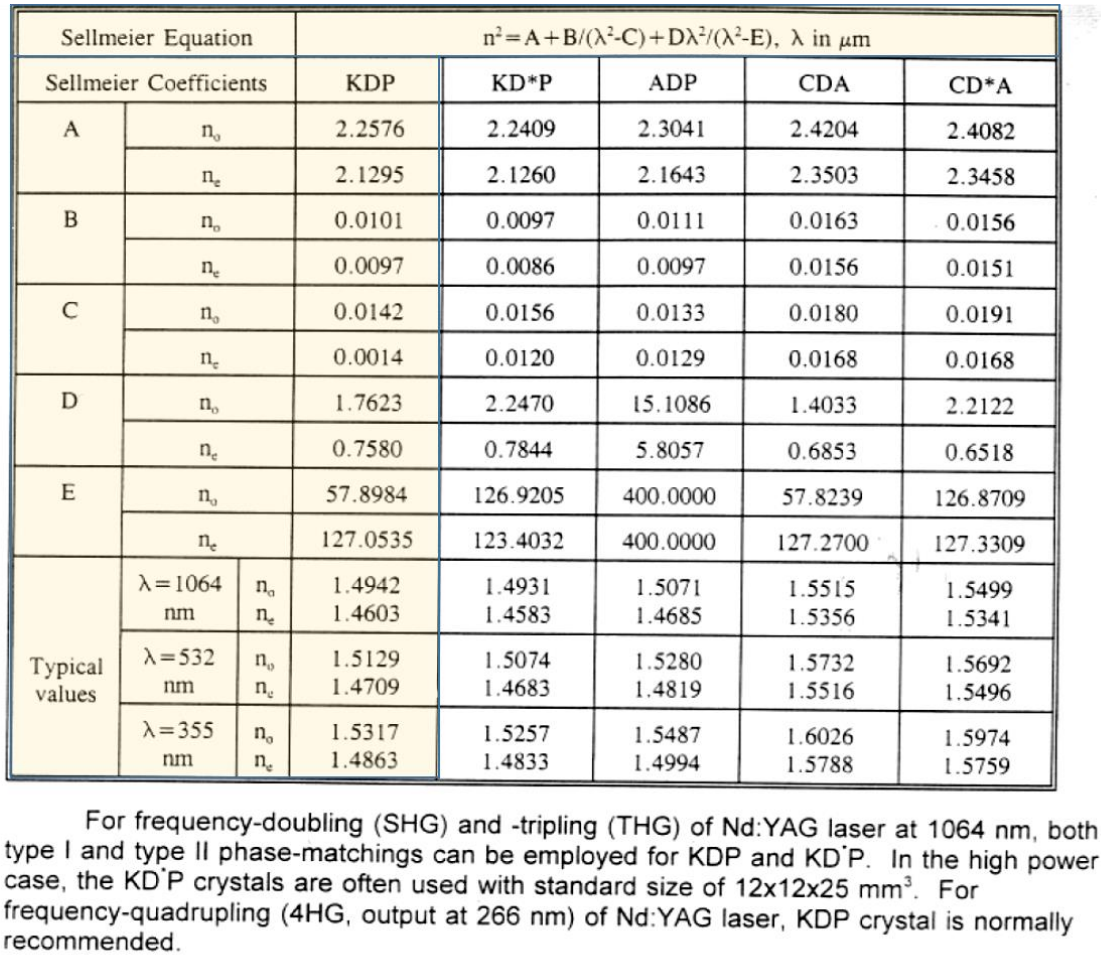
(b) Now make the *low-depletion* approximation by setting $|A_1|^2=|A_1(0)|^2$ in the above equation. Solve for A_1 for a propagation length L. (Hint: You need a second initial condition that is obtained from $E_2(0)=0$).

(c) Taking $A_1=|A_1|e^{i\phi}$, plot $|A_1|/|A_1(0)|$ and ϕ versus ΔkL (from -4π to 4π) for Γ^2L^2=0.1, 0.2, and 0.4. Discuss your results (i.e. sign reversal vs. Δk, etc.)

The above process (i.e. the intensity-dependent phase variation of the fundamental wave) has been termed $\chi^{(2)}:\chi^{(2)}$ cascading nonlinearity. It mimics a third order $\chi^{(3)}$ process where $\chi^{(3)}_{eff}\propto\chi^{(2)}(\omega;2\omega,-\omega)\chi^{(2)}(2\omega;\omega,\omega)$ is effectively a cascade of two second order effects. The cascading nonlinearity has generated some interest for applications requiring large $\chi^{(3)}$ effects (i.e. optical switching, spatial solitons, and, in general, processes requiring an n_2-type nonlinearity). See Sheik-Bahae and Hasselbeck (OSA Handbook, Chapter 17).

Problem 4. Cascading for THG (Third-Harmonic Generation) in KDP.
Actually, cascading 2nd order effects to obtain an effective third-order effect is not a new concept. In fact the most efficient way to generate the third-harmonic (3ω) of a laser beam is to first produce 2ω (in an SHG process) and then use SFG to generate $3\omega=2\omega+\omega$. The phase matching requirement, however, dictates that this *cascading* be performed in two separate crystals with proper orientation. In Problem 1, you calculated the phase matching angle (ϕ_m) for type-I SHG in KDP. Now calculate ϕ_m for a second crystal to produce the third-harmonic of a YAG laser. (Note: No rotation of polarization is used between the two crystals).





Sellmeier Equation			$n^2=A+B/(\lambda^2-C)+D\lambda^2/(\lambda^2-E)$, λ in μm				
Sellmeier Coefficients			KDP	KD*P	ADP	CDA	CD*A
A	n_o		2.2576	2.2409	2.3041	2.4204	2.4082
	n_e		2.1295	2.1260	2.1643	2.3503	2.3458
B	n_o		0.0101	0.0097	0.0111	0.0163	0.0156
	n_e		0.0097	0.0086	0.0097	0.0156	0.0151
C	n_o		0.0142	0.0156	0.0133	0.0180	0.0191
	n_e		0.0014	0.0120	0.0129	0.0168	0.0168
D	n_o		1.7623	2.2470	15.1086	1.4033	2.2122
	n_e		0.7580	0.7844	5.8057	0.6853	0.6518
E	n_o		57.8984	126.9205	400.0000	57.8239	126.8709
	n_e		127.0535	123.4032	400.0000	127.2700	127.3309
Typical values	λ=1064 nm	n_o	1.4942	1.4931	1.5071	1.5515	1.5499
		n_e	1.4603	1.4583	1.4685	1.5356	1.5341
	λ=532 nm	n_o	1.5129	1.5074	1.5280	1.5732	1.5692
		n_e	1.4709	1.4683	1.4819	1.5516	1.5496
	λ=355 nm	n_o	1.5317	1.5257	1.5487	1.6026	1.5974
		n_e	1.4863	1.4833	1.4994	1.5788	1.5759

For frequency-doubling (SHG) and -tripling (THG) of Nd:YAG laser at 1064 nm, both type I and type II phase-matchings can be employed for KDP and KD*P. In the high power case, the KD*P crystals are often used with standard size of 12x12x25 mm^3. For frequency-quadrupling (4HG, output at 266 nm) of Nd:YAG laser, KDP crystal is normally recommended.