Problem 1. Two optical beams $E_1$ and $E_2$ with wavelengths of 1.0 and 0.6 $\mu$m respectively are incident on a nonlinear material.

![Diagram of E1 and E2 beams](image)

(a) Assuming a $\chi^{(2)}$ nonlinearity, what new wavelengths can possibly be generated in this material?

The above nonlinear material is now replaced with a centro-symmetric material for the remaining part of this problem.

(b) What is the dominant nonlinear susceptibility?

(c) Assuming $\chi^{(3)}$ nonlinearity, what new wavelengths $\lambda_j$ can possibly be generated in this material that simultaneously involve the interaction of both $E_1$ and $E_2$ beams? Write down the corresponding nonlinear polarization $P(\lambda_j)$ including their $\chi^{(3)}(\lambda_j; \lambda_k, \lambda_q, \lambda_p)$ terms (ignore Cartesian indices).

(d) If $|E_1| >> |E_2|$, identify the most dominant terms in part (c).

(e) Write down the nonlinear polarization terms associated with self- and cross phase modulation of each beam (identify $\chi^{(3)}(\lambda_j; \lambda_k, \lambda_q, \lambda_p)$ terms)

(f) Under what condition the simultaneous presence of both beams leads to a nonlinear attenuation (absorption) of both beam? Describe this process, the required energy resonance (use diagrams), and the nature of the complex susceptibility $\chi^{(3)}(\lambda_j; \lambda_k, \lambda_q, \lambda_p)$ (with respect to part e).

(g) Under what condition the simultaneous presence of both beams leads to a nonlinear attenuation (absorption) of one beam (which?) and gain in the other (which?)? Describe this process, the required energy resonance (use diagrams), and the nature of the complex susceptibility $\chi^{(3)}(\lambda_j; \lambda_k, \lambda_q, \lambda_p)$ (with respect to part e and f).
Problem 2. Two-Photon Spectroscopy:

The 1S-2S transition in atomic Hydrogen (E=10.206 eV) is investigated using two-photon spectroscopy with two narrow-band CW laser sources. A pump laser with fixed wavelength $\lambda_1=200$nm and a tunable laser ($\lambda=250$-$350$ nm) are used in a counter propagating arrangement as shown.

(a) Qualitatively plot the transmission of the probe beam as a function of its tunable wavelength $\lambda_2$.

(b) Will the result in (a) be any different if they two beams were co-propagating? (Hint: think Doppler!)