NONLINEAR OPTICS (PHYC/ECE 568)

Spring 2022 - Instructor: M. Sheik-Bahae

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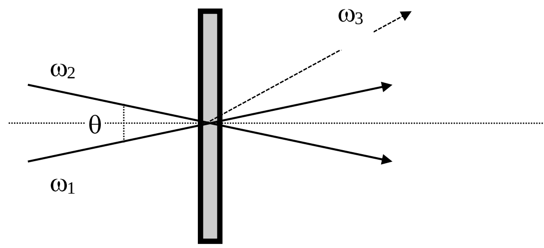
Homework #7, Due Wed. Apr. 14

1. Nearly Degenerate FWM

Consider the FWM (forward) geometry shown in the figure below where two pump beams at frequency ω_1 and ω_2 ($\omega_1=\omega_2+\delta$, δ small) are incident on a $\chi^{(3)}$ material to produce a signal at a frequency $\omega_3=2\omega_1-\omega_2$. For very small angles θ, what is direction (i.e. the angle) with which the generated signal at ω_3 will be propagating? Note: δ is small enough to ignore dispersion.



2. Self-Phase Modulation (SPM)

a. A laser pulse with an intensity profile $I=I_0\mathrm{Sech}^2(t/\tau_0)$ having τ_0=200 fs and λ=500 nm, and I_0=1 GW/cm^2 is coupled into a silica fiber having an instantaneous (ultrafast) $n_2=2\times10^{-16}$ cm^2/W. **Estimate** the required length of fiber (L) for the spectrum of the pulse to broaden (due to SPM only) to ≈5 times its original value.

b. Using linear dispersive elements such as grating pairs, we can compress the exit pulse in part (a) to a transform-limited pulse having a width $\approx 1/\Delta\omega$. **Estimate** the resultant compressed pulse width.

c. If the pulse in (a) propagates in a nonlinear medium with $n_2\approx1\times10^{-14}$ cm^2/W but with a relaxation time τ=2 ps (i.e. nonlinear refraction accumulates) , what is the required length to double its spectrum. Qualitatively, describe the transmitted spectrum as compared to that of part (a).

3. Pick one of the following two problems (next 2 pages) on Intensity Dependent Refractive Index:

3(a) Beam Deflection

As briefly discussed in the class, among the sensitive methods of nonlinear refraction measurements is the beam deflection technique. As shown in the figure, a pump beam (with a Gaussian spatial profile) induces a refractive index profile in the nonlinear material. A weak probe beam-with a diameter much smaller than that of the pump- centered at a distance x_0 away from the pump beam will be therefore deflected by this index gradient.

Assume a pump irradiance $I(r)=I_0 exp(-2r^2/w_e^2)$ and optical Kerr effect: $n=n_0+n_2 I$. We also know that a ray traveling through a thin material having a transverse gradient index $n(r)$ is deflected by an angle $\phi = \nabla n(r) L/n_0$ where L is the thickness of the sample and n_0 is the linear refractive index.

a) What is the optimum position x_0 for which the maximum deflection occurs?

b) What is the maximum deflection angle?

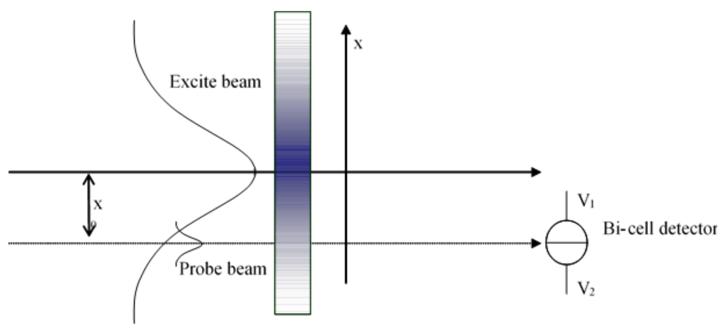
The deflection of the beam is measured using a "bi-cell" detector in the *far field* as shown in the figure. In that case, the measured normalized signal is $S=(V_1-V_2)/(V_1+V_2)$.

c) Show that

$$S = \sqrt{8\pi}\,\frac{w_0}{\lambda}\phi$$

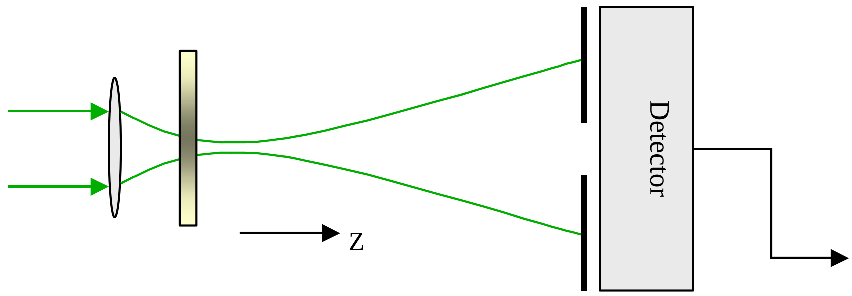
where w_0 the probe beam radius at the sample and λ is the wavelength.

d) Write S in terms of the maximum on axis phase shift $\Delta\Phi_0=(2\pi/\lambda)n_2 I_0 L$ (i.e. $S=K\Delta\Phi_0$). Derive an expression for K and compare the sensitivity of this technique with that of z-scan where the normalized peak-to-valley transmittance signal is given by $\Delta T_{pv} \approx 0.4\Delta\Phi$ for $w_0 \approx w_e$, $n_0 = 2$.



3(b) Z-Scan (See also Problem 7.3 in Boyd)

A thin nonlinear optical material having a thickness L and a nonlinear index coefficient n_2 is scanned along z (propagation direction) near the focus of a Gaussian beam that is characterized by its minimum spot size w_0, wavelength λ_0 and power P (see Figure). We know, from aberration-free approximation, that the induced Kerr-lens focal length $f_{nl}(z) = aw^2(z)/4Ln_2I(z)$ where $I(z)=2P/\pi w^2(z)$ is the on-axis intensity , $w(z)^2 = w_0^2(1+z^2/z_0^2)$, and ***a*** is a numerical correction factor.



(a) Use the ABCD matrix formalism to derive expressions for the beam radius w_a and on-axis intensity I_a ($=2P/\pi w_a^2$) at a distance *d* from the focus.

(b) Simplify the expression for I_a (obtained in (a)) by retaining the lowest order nonlinear term in the power expansion and by assuming that $d>>Z,Z_0$. You should be able to express your results in terms of $x=z/z_0$ and $\Delta\Phi_0/a$ where $\Delta\Phi_0=(2\pi/\lambda_0)n_2I(0)L$ is the on-axis nonlinear phase shift at the focus.

(c) By placing a small on-axis aperture and a detector at the observation plane we can measure I_a. Obtain an expression for the normalized transmittance $T(x, \Delta\Phi_0)= I_a(\Delta\Phi_0,x)/I_a(0,x)$. Plot T versus x for $-4<x<4$.

(d) Compare your results in (c) with that obtained using the diffraction theory:

$$T(\Delta\Phi_0, x) \cong 1 + \frac{4x\Delta\Phi_0}{(1+x^2)(9+x^2)}$$

What value of **a** (the correction factor) gives a good overall agreement between the two approaches.