Nonlinear Optics (PHYC/ECE 568)
Spring 2007

Midterm Exam I, Open Text-Book, Time: 1.5 hour

NAME ....
last *first*

Grade

Please staple and return this page with your exam.

1. SHG Bandwidth (a HW problem):

a. Calculate the bandwidth $\Delta\omega$ associated with a phase-matched SHG process in terms of the group velocities $v_g(\omega_l)$ and $v_g(2\omega_l)$. *Bandwidth is defined as the frequency spread around* ω_l, *for which* $\Delta k(\omega_l \pm \Delta\omega/2)L = 2\pi$. with L denoting the length of the nonlinear crystal. *Hint: Use the first-order term in the Taylor series expansion of* $\Delta k(\omega)$. *(15 pts.)*

b. Discuss how your results in (a) explains the limitation on the SHG-efficiency when ultrashort laser pulses are used.

(5 pts.)

Note group velocity $v_g = (dk/d\omega)^{-1}$

2. SHG Acceptance Angle:

a. In a similar manner as in problem 1, calculate the acceptance angle ($\Delta\theta$) for a type-I phase matched SHG process. A*cceptance angle is defined as angular spread around* θ_m, *for which* $\Delta k(\theta_m \pm \Delta\theta/2)L = 2\pi$. Assume negative birefringence in a uniaxial crystal where $\Delta k(\theta) = (2\omega/c)(n_e^{2\omega}(\theta) - n_o^{\omega})$. *(20 pts.)*

b. Assuming small birefringence ($n_e \approx n_o$), show that

$$\Delta\theta \cong \frac{\lambda_\omega}{L(n_o^{2\omega} - n_e^{2\omega})\sin(2\theta_m)} \quad (5\ pts.)$$

c. What limitation does this impose on the useful length of the crystal for an incident focused Gaussian beam of waist w_0 (recall: divergence angle is $\lambda/\pi n w_0$). *(10 pts.)*

d. Briefly discuss the implications of the above relationship for various angles (e.g. for $\theta_m = 90^o$). *(5 pts.)*

3. $\mathbf{d_{eff}}$:

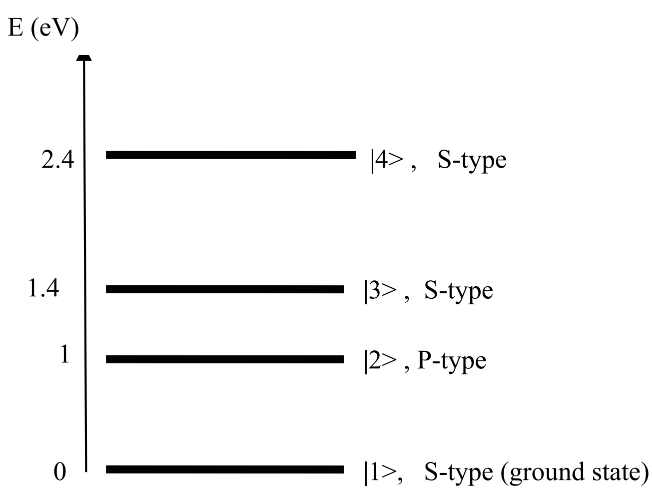
Consider $AgGaSe_2$ crystal in a SHG process. The orientation of the input field at ω is given by $E(\omega) = A_0(2\hat{x} + 1\hat{y} + 3\hat{z})$, where x, y, and z are the principle axes of the crystal. What is d_{eff}? *(10 pts.)*

Note: From table 1.5.3 and Fig. 1.5.2 (in Boyd) you get d_{il} (pm/V) tensor in $AgGaSe_2$ to be:

$$\begin{pmatrix} 0 & 0 & 0 & 81 & 0 & 0 \\ 0 & 0 & 0 & 0 & 81 & 0 \\ 0 & 0 & 0 & 0 & 0 & -81 \end{pmatrix}$$

4. A fictional molecule has the following energy levels. Draw the spectrum (for $0<\hbar\omega<3$ eV) for the **(a)** linear absorption coefficient α, **(b)** two-photon absorption coefficient β, **(c)** $|\chi^{(2)}(2\omega;\omega,\omega)|$ and **(d)** $|\chi^{(3)}(3\omega;\omega,\omega,\omega)|$. *(30 pts.)*

Be quantitative in your x-axis. Assume a finite broadening in your drawings. Point out the resonances (diagrammatically) on your graph for each case and show the relative strengths if obvious. Note: no calculations needed for this problem.



1- See H.W Solutions

2- SHG B.W

$n_o > n_e$

$$\Delta k(\theta) = \frac{2\omega}{c}\left(n_e^{2\omega}(\theta) - n_o^{\omega}\right)$$

Taylor Series Expansion

$$\Delta k(\theta) = \cancel{\Delta k(\theta_m)}_{0} + \left.\frac{\partial k}{\partial \theta}\right|_{\theta=\theta_m}(\theta-\theta_m) + (\theta-\theta_m)^2 + \cdots$$

$$\Delta k\left(\theta_m + \frac{\Delta\theta}{2}\right)L = \left.\frac{\partial k}{\partial \theta}\right|_{\theta_m}\frac{\Delta\theta}{2}L = 2\pi$$

$$\Delta\theta = \frac{4\pi}{\left(\frac{\partial k}{\partial \theta}\right)_{\theta_m} L}$$

$$\frac{\partial k}{\partial \theta} = \frac{2\omega}{c}\frac{\partial n_e^{2\omega}(\theta)}{\partial \theta}$$

$$\left(n_e^{2\omega}(\theta)\right)^{-2} = \left(n_e^{2\omega}\right)^{-2}\sin^2\theta + \left(n_o^{2\omega}\right)^{-2}\cos^2\theta$$

$$n_e^{2\omega}(\theta) = \left[\left(n_e^{2\omega}\right)^{-2}\sin^2\theta + \left(n_o^{2\omega}\right)^{-2}\cos^2\theta\right]^{-\frac{1}{2}}$$

$$\left.\frac{\partial n_e^{2\omega}}{\partial \theta}\right|_{\theta=\theta_m} = -\frac{1}{2}\left[\frac{2\sin\theta_m\cos\theta_m}{(n_e^{2\omega})^2} + \frac{-2\sin\theta_m\cos\theta_m}{(n_o^{2\omega})^2}\right]\left[\quad\right]^{-3/2}$$

$$= -\frac{\sin 2\theta_m}{2}\left[\frac{1}{(n_e^{2\omega})^2} - \frac{1}{(n_o^{2\omega})^2}\right] \cdot \underbrace{\left[\qquad\right]^{-3/2}}_{(n_o^{\omega})^3}$$

$$\frac{\partial n_e(\theta)}{\partial \theta} = \frac{-\sin 2\theta_m}{2} \times (n_o^{\omega})^3 \left(\frac{1}{(n_e^{2\omega})^2} - \frac{1}{(n_o^{2\omega})^2}\right).$$

(b)

$$n_e \simeq n_o \simeq n \qquad \Delta n$$

$$\frac{\partial n_e(\theta)}{\partial \theta} \simeq -\frac{\sin 2\theta_m}{2} \times \frac{n^3}{n^4} (n_o^2 - n_e^2)^{2\omega} = \Delta n$$

$$\simeq -\frac{\sin 2\theta_m}{2} \times \frac{n^3}{n^4}\, 2n\,\Delta n$$

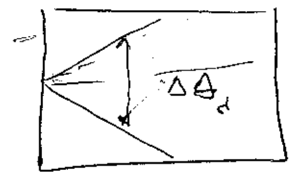
$$\Delta n \equiv (n_o - n_e)_{2\omega}$$

$$\simeq \sin 2\theta_m \;\Delta n.$$

$$\left.\frac{\partial k}{\partial \theta}\right|_{\theta_m} = \frac{2\omega}{c} \sin 2\theta_m\, \Delta n = \frac{4\pi\,\Delta n \sin \theta_m}{\lambda_\omega}$$

$$\Delta\theta = \frac{\lambda_m}{\Delta n \sin\theta_m \times L}$$

(c)



$$\Delta\theta_{div} = \frac{\lambda}{\pi n W_0}$$

We must have

$$\Delta\theta_{div} < 2\Delta\theta$$

(Note: $\Delta\theta_{div}$ is half-angle, $\Delta\theta$ is full angle by definition)

$$\frac{\lambda_\omega}{\pi n W_0} < \frac{2\lambda_\omega}{\Delta n \sin 2\theta_m \cdot L}$$

$$L < \frac{2\pi n W_0}{\Delta n \sin 2\theta_m} = l_d$$

(Note this is almost the same as l_a from Poynting vector walk-off). (HW#3)

$$l_a = \frac{\sqrt{\pi}\, n W_0}{\Delta n \sin 2\theta_m}$$

(d) at $\theta_m = \frac{\pi}{2}$, $l_d \to \infty$!

This is because $\frac{\partial k}{\partial \theta} \to 0$ and we must take higher order terms in the expansion into account. $\left(\frac{\partial^2 k}{\partial \theta^2}, \ldots\right)$.

This leads to $\Delta\theta = \sqrt{\dfrac{\lambda_\omega}{2L(n_o^{2\omega} - n_e^{\omega})}}$

(3)

$$E(\omega) = A_0 (2\hat{x} + 1\hat{y} + 3\hat{z})$$

$$= \underbrace{A_0 \times \sqrt{14}}_{E_0} \underbrace{\left(\frac{2}{\sqrt{14}} \hat{x} + \frac{1}{\sqrt{14}} \hat{y} + \frac{3\hat{z}}{\sqrt{14}} \right)}_{\hat{e} \text{ unit vector.}}$$

$$P = 2 d_{eff} |E_0|^2$$

$$= 2 \begin{bmatrix} 0 & 0 & 0 & 81 & 0 & 0 \\ 0 & 0 & 0 & 0 & -81 & 0 \\ 0 & 0 & 0 & 0 & 0 & -81 \end{bmatrix} \begin{bmatrix} E_x^2 \\ E_y^2 \\ E_z^2 \\ 2E_yE_z \\ 2E_xE_z \\ 2E_yE_x \end{bmatrix}$$

$$\begin{bmatrix} \frac{4}{14} \\ \frac{1}{14} \\ \frac{9}{14} \\ \frac{6}{14} \\ \frac{12}{14} \\ \frac{4}{14} \end{bmatrix} |E_0|^2$$

$$d_{eff} = 81 \left[\frac{6}{14} - \frac{12}{14} - \frac{4}{14} \right] = -\frac{810}{14} \text{ Pm/V}$$

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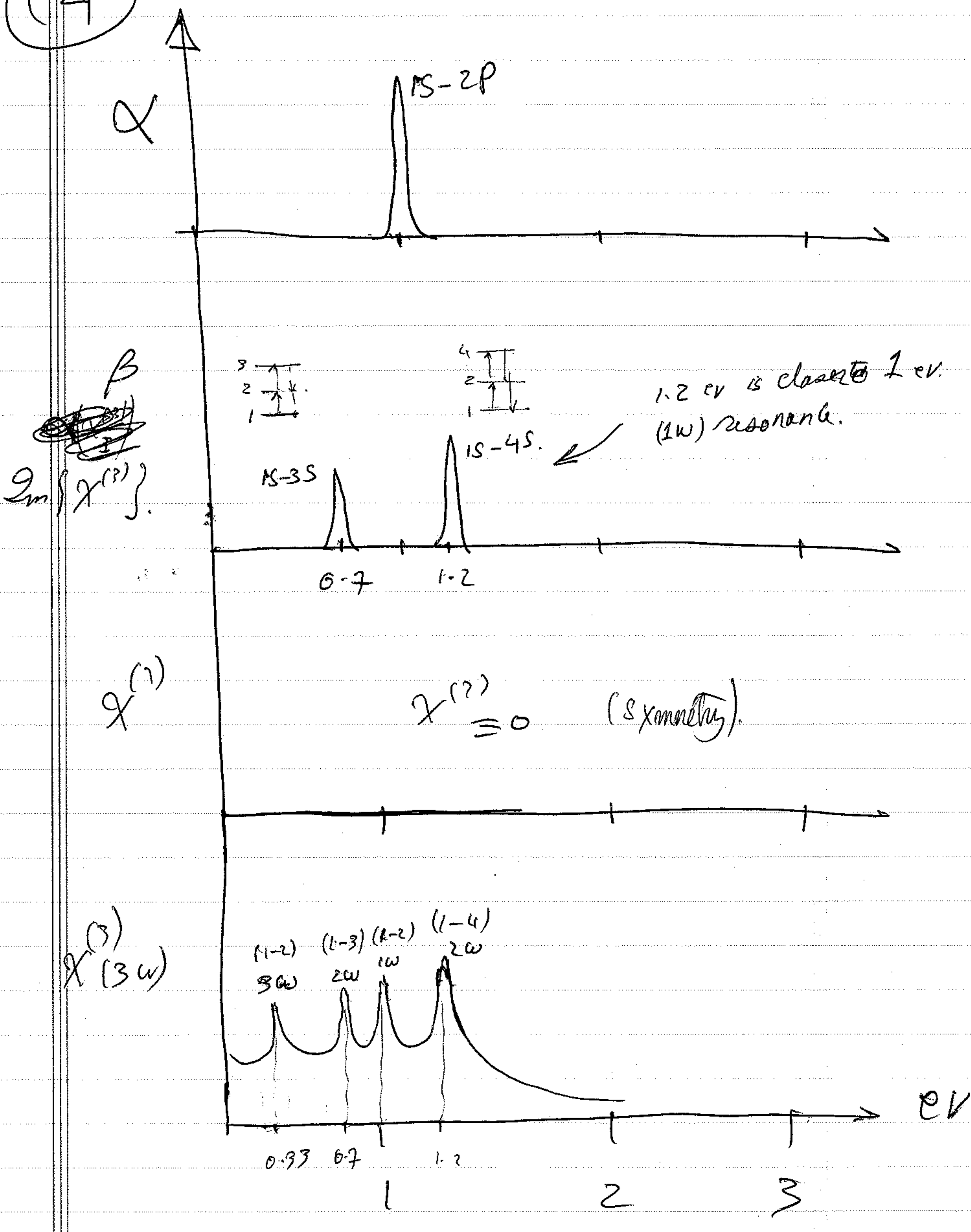
α
1S-2P
β
Im{χ(3)}
3
2
1
4
2
1
1S-3S
1S-4S.
1.2 eV is closer to 1 eV.
(1ω) resonance.
0.7
1.2
χ(2)
χ(2) ≡ 0 (symmetry).
χ(3)(3ω)
(1-2) 3ω
(1-3) 2ω
(1-2) 1ω
(1-4) 2ω
0.33
0.7
1.2
1
2
3
eV