3 Problems

1. Verdeyen: Problem 8.26

8.26. The spontaneous emission from a certain (semiconductor) laser can be approximated by the sketch shown below. The spontaneous lifetime is 5 ns.

(a) What is the stimulated emission cross section?
(b) If the facet (mirror) reflectivities (for power) were 0.32 and the length of the medium completely filling the cavity were 450 μm, then what must be the inversion density to obtain oscillation?
(c) What is the photon lifetime $\tau_p$?

2. Verdeyen: Problem 8.31

A fiber glass amplifier for 1.55 μm radiation has a small signal gain of 23 dB (i.e., 200X), a stimulated emission cross section of $10^{-16}$ cm$^2$, an upper state lifetime of 130 μs, and a very short lower state lifetime. Assume steady state, equal degeneracies, and that the lifetime of state 1 is very short.

(a) If the length of this fiber amplifier were 100 cm, what must be the inversion $N_2 - N_1$ to obtain this small signal gain?
(b) How much reflection at the input and output of this amplifier can be tolerated before it breaks into oscillation?
(c) At what value of the input intensity will the gain of this amplifier be 20 dB (i.e., 100X)?
3. Verdeyen: Problem 9.11

Consider an optically pumped laser system depicted in the diagram below. A strong optical pump tuned to line center of the $0 \rightarrow 2$ transition is incident on the sample from the side and promotes atoms from state 0 to 2. The atoms in state 2 can decay back to 0 by the indicated spontaneous emission ($A_{21} = 10^8$ sec$^{-1}$) and/or quenching processes ($k_{20} = 5 \times 10^6$ sec$^{-1}$), or to state 1 by spontaneous emission ($A_{20} = 10^6$ sec$^{-1}$) and/or stimulated emission. The spacing $E_2 - E_0$ is much larger than $kT$, and thus we can neglect any initial population in state 1. Atoms in state 1 find their way back to 0 at a rate $r_1^{-1} = 10^7$ sec$^{-1}$. To make the problem simple, assume that the density of state 2 (or 1) is always much less than that of state 0, and thus we need not worry about the conservation of atoms (or a rate equation for state 0), that is, $N_0 = \text{constant}$. Assume steady state and a homogeneous line shape for all transitions with $\Delta \nu_k = 10$ GHz.

(a) What is the absorption cross section for the pump wave at the frequency $\nu_{20}$?

(b) What is the stimulated emission cross section for the $2 \rightarrow 1$ transition?

(c) What is the lifetime of state 2?

(d) If the optical pump intensity is weak, the $2 \rightarrow 1$ transition does not lase and stimulated emission at $\nu_{21}$ can be neglected. Under these circumstances, what is the ratio of the population densities $N_2/N_1$?

(e) What is the minimum inversion density, $N_2 = (g_2/g_1)N_1$, required to achieve oscillation on the $2 \rightarrow 1$ transition?

(f) Formulate the steady state rate equation for states 2 and 1 in terms of $\sigma_p$, $I_p$, $\sigma_{21}$, and $I_{21}$, and the appropriate atomic relaxation rates. Use these equations to determine the intensity of the pump to reach threshold on the $2 \rightarrow 1$ transition.

(g) If the pump is 10 times the value required to reach threshold at $\nu_{21}$, then the stimulated emission rate on the $2 \rightarrow 1$ transition overwhelms all other loss processes for state 2 and pumping processes for state 1. Use this fact to neglect the appropriate terms in the rate equations and thus predict the output intensity of this laser. (The approximations are intended to simplify matters, but if they confuse the issue, do it the long way. It is not that much more difficult.)

(h) If the pump intensity is very large, then the approximation that the ground state density does not change is not valid. The atoms are pumped to state 2, immediately stimulated to make a transition to 1, and then slowly relax back to 0. By using this scenario, predict the pump intensity at which the ground state density is depleted by 30%.