8.2. An experiment involving a homogeneously broadened optical amplifier is depicted in the diagram below. For an input intensity of 1 W/cm², the gain (output/input) is 10 dB. If the input intensity is doubled to 2 W/cm², the gain is reduced to 9 dB.

(a) What is the small-signal gain (i.e., $I_{in} \rightarrow 0$) of this amplifier (in dB)?
(b) What is the saturation intensity?
(c) What is the maximum power (per unit area) that can be extracted from this amplifier (in limit of large input intensity)?
(d) What must be the input intensity to extract 50% of this maximum?

8.5. The ideas of gain saturation are equally applicable to absorption, and that is the purpose of this problem. Consider a single-frequency dye laser tuned to the center of the sodium D line at 5889.95 Å and irradiating a heated cell (630³K), 10 cm long, containing a mixture of sodium (Na) vapor at a density of $1.5 \times 10^{15}$ cm⁻³ and helium (He) gas at a density of $6.53 \times 10^{19}$ cm⁻³. The self-broadening of this line, caused by collisions between sodium atoms, is 15 MHz for the conditions of this problem. The foreign gas broadening is due to collisions between sodium atoms and helium with a cross section estimated to be $10^{-14}$ cm². The following data for this transition are from NSRDS-NBS (vol. 11), U.S. Department of Commerce, National Bureau of Standards:

$$N_1 \quad 3^2 S_{1/2} \quad g_1 = 2 \quad E_1 = 0$$
$$N_2 \quad 3^2 P_{3/2} \quad g_2 = 4 \quad E_2 = 16,978.07 \text{ cm}^{-1}; \quad A_{21} = 6.3 \times 10^7 \text{ sec}^{-1}$$

(a) What are all of the pertinent line widths from the various causes ("natural," Doppler, self-broadening, or foreign gas [He] collisions)?
(b) If a "small-signal" laser is tuned to line center and propagates through the 10 cm length, what fraction emerges? Express the attenuation in dB.
(c) Let the input amplitude of the laser be a variable. Plot the transmission (in percent of the incident value) as a function of the input intensity normalized to a saturation value similar to that shown in Fig. 8.6. To find $I_1$, follow the procedure of Sec. 8.3, but remember that $N_1 + N_2 = [Na]$ (i.e., the total number of sodium atoms is conserved).
8.7. Consider the ideal laser medium shown below. The pump excites the atoms to state 2 at a rate \( R_2 \), which then decays to state 1 at a rate \( \tau_{21}^{-1} \) and back to state 0 at a rate \( \tau_{20}^{-1} \). State 1 decays back to 0 so fast that the approximation \( N_1 \approx 0 \) is appropriate. The radiative rate for the \( 2 \rightarrow 1 \) transition is \( 6 \times 10^6 \) sec\(^{-1} \), and its width is 10 GHz. (Assume a Lorentzian profile and steady state.)

(a) What is the stimulated emission cross section?

(b) What must be the pump rate \( R_2 \) in order to obtain a small-signal gain coefficient of 1%/cm?

(c) What is the saturation intensity for the \( 2 \rightarrow 1 \) transition?

(d) How much power (in W/cm\(^2\)) is expended in creating the gain coefficient of (b)?

\[ \begin{align*}
5.5 \text{ eV} & \quad \downarrow \quad 2 \\
3.2 \text{ eV} & \quad \downarrow \quad 1 \\
& \quad \downarrow R_2 \\
& \quad \downarrow 0 \\
\end{align*} \]

\[ A_{21} = 6 \times 10^6 \text{ sec}^{-1} \]
\[ \tau_{21} = 100 \text{ ns} \]
\[ \tau_{20} = 200 \text{ ns} \]

(e) Express the line width in Å units and cm\(^{-1}\) units.