1. Consider a pressure-broadened gaseous two-level medium with the following property:
   - Spontaneous emission lifetime: $\tau_{sp} = 1 \mu s$
   - Homogeneous linewidth $\Delta \nu_h = 1.5$ THz
   - Line center wavelength: $\lambda_0 = 5 \mu m$
   - Molecular density (concentration): $N_{total} = 2.5 \times 10^{19} \text{ cm}^{-3}$
   - Non-degeneracy factors: $g_1 = 5$, $g_2 = 1$

   (a) What is the absorption coefficient $\alpha (\text{cm}^{-1})$ at the line center (5 $\mu m$) when all the molecules are in their ground state (level 1)?

   (b) What fraction of the molecules needs to be excited into level 2 in order to make this gas transparent (i.e. the onset of gain) at 5 $\mu m$?

2. 7.3. The spontaneous emission profile from a certain transition can be approximated by the shape shown below.

(a) What is the stimulated emission cross section?
(b) What is the absorption cross section?
7.5. Consider a transition of 5000 Å with a width of 1 Å, a cavity of 2 cm$^3$ in volume and let $n = 1$.

(a) Convert this wavelength interval (1 Å) to frequency units (i.e., GHz and cm$^{-1}$).
(b) How many electromagnetic modes exist in this frequency band for this cavity?
(c) Suppose that the cavity were in the form of a cylinder with a cross-sectional area of 0.1 cm$^2$ (and thus is 20 cm long). How many TEM$_{0,0,q}$ cavity modes would fit within the frequency band specified by this 1 Å? (Do not forget the two polarizations.)
(d) Combine the results of (b) and (c) to estimate the probability of a spontaneous photon appearing in one of the polarized TEM$_{0,0,q}$ modes.
(e) If the $A$ coefficient for this transition is $10^7$ sec$^{-1}$, what is the stimulated emission cross section?

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7.11. The spontaneous emission profile of a certain laser can be approximated by the triangular shape shown below. If the spontaneous lifetime were 5 nsec and the gain coefficient were 10 cm$^{-1}$, find

(a) The value of the line shape (in sec) at $h\nu/e = 1.476$ eV
(b) The inversion necessary to obtain that gain coefficient
Problem 1

\[ \tau_{\text{spont}} := 1 \cdot 10^{-6} \, \text{s} \quad \Delta \nu_\text{h} := 1.5 \cdot 10^{12} \, \text{Hz} \quad \lambda_0 := 5 \cdot 10^{-6} \, \text{m} \]

\[ N_t := 2.5 \cdot 10^{19} \, \text{cm}^{-3} \quad g1 := 5 \quad g2 := 1 \quad n := 1 \]

\[ A_{21} := \frac{1}{\tau_{\text{spont}}} \quad g0 := \frac{1}{\Delta \nu_\text{h}} \]

(a)

Cross Section:

\[ \sigma_0 := A_{21} \frac{\lambda_0^2}{8 \pi \cdot n^2} \cdot g0 \]

\[ N2 := 0 \quad N1 := N_t \]

\[ \gamma_0 := \sigma_0 \left( N2 - \frac{g2}{g1} \cdot N1 \right) \]

\[ \alpha_0 := -\gamma_0 \]

\[ \alpha_0 = 3.316 \times 10^6 \, \frac{1}{\text{m}} \]

(b) For transparency (\( \gamma = 0 \)) we have \( FR \cdot N_t \cdot g2 / g1 (1 - FR) \cdot N_t = 0 \) where \( FR \) is the fraction of population that is excited to level 2.

\[ FR := \frac{g2}{g1} \frac{g2}{1 + g2/g1} \]

\[ FR = 0.1 \quad \text{Fraction: 10 \%} \]
The value of $\overline{\nu}$ is $\overline{\nu} = 18,340 - 2,627 = 15,713 \text{ cm}^{-1}$ and it is centered at zero in $g(\nu)$'s graph.

\[ \int g(\nu) d\nu = 1 \Rightarrow \left( \frac{K}{3} \cdot \frac{1}{2} + (1/3 + k) \cdot \frac{3}{2} + \frac{K}{2} \right) \text{ cm}^{-1} = 1 \]

\[ \Rightarrow \frac{8K}{3} \text{ cm}^{-1} = 1 \Rightarrow K = \frac{3}{8} \text{ cm} \]

\[ \overline{\nu} = \frac{1}{\gamma} \]

Now \[ \gamma = \frac{c}{\nu} \Rightarrow \overline{\nu} = \frac{c}{\gamma} = \frac{3 \times 10^8}{10^{-2} \text{ m}} = 3 \times 10^{10} \text{ Hz} \]

Thus \[ 1 \text{ cm}^{-1} = 3 \times 10^{10} \text{ Hz} \]

\[ \Rightarrow K = \frac{3}{8} \cdot \frac{1}{3 \times 10^{10}} = 1.25 \times 10^{-11} \text{ sec.} = g(\overline{\nu}) \]

We have: \[ g(\nu) = A_2 \frac{\nu^2}{8 \sin^2 \theta} \]
\[ \lambda_0 = \frac{1}{15.719} \text{ cm} = 636 \times 10^{-9} \text{ m} \]

\[ \Rightarrow G(\theta) = \frac{10 (636 \times 10^{-9})^2}{8 \pi} \times 1.25 \times 10^{-11} = 2.011 \times 10^{-24} \text{ m}^2 \]

\[ = 2.011 \times 10^{-20} \text{ cm}^2 \]

(b) Using \[ g_2(\theta) = 2 J_2(\theta) + 1 \Rightarrow g_4 = 5 \text{ & } g_2 = 3 \]

Now \[ G_{\text{abs}} = \frac{g_2}{g_1} G_{\text{SE}} \]

\[ \Rightarrow G_{\text{abs}} = \frac{3}{5} \times 2.011 \times 10^{-20} = 1.2 \times 10^{-20} \text{ cm}^2 \]
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(a) \(0.080 \text{ ev} = \hbar \Delta \nu\)
\[\Delta \nu = \frac{0.08 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-34}} = 1.933 \times 10^7 \text{ Hz}\]

\(1.476 \text{ ev} = \hbar \nu_0 \quad \Rightarrow \quad \nu_0 = 3.56 \times 10^7 \text{ Hz}\)

\[\mathcal{J} \left( \phi \right) \text{ d} \phi = 1 \quad \Rightarrow \quad \frac{1}{2} \left( 1.933 \times 10^7 \times H \right) = 1 \quad \Rightarrow \quad H = \frac{2}{1.933 \times 10^7} = 1.03 \times 10^{-13} \text{ sec}\]

(b) We know that:
\[\frac{dI}{dz} = \left( A_2 \frac{\lambda^2}{8 \pi n^2 q \lambda_0^2} \right) \left( N_2 - \frac{q_2}{q_1} M_1 \right) \frac{\Delta N}{\lambda_0} \]
\[\lambda_0^2 = 10 \, \text{cm}^{-1}\]

\[\Rightarrow \quad \lambda_0 = \frac{c}{\nu_0} = 842.7 \, \text{nm}\]

\[\Rightarrow \quad N = \left( \frac{10}{\text{cm}^3} \right) \frac{8 \pi n^2}{A_2 \lambda^2 q \lambda_0^2} = \frac{3^3 \times 8 \pi \times 1}{5 \times 10^9 \left( 842.7 \times 10^{-9} \right)^2 \times 1.03 \times 10^{-13}} \]

\[\Rightarrow \quad N = 1.72 \times 10^{21} \, \text{m}^{-3}\]
$\lambda_0 = 5000 \text{ Å}$

$V = 2 \text{ cm}^3$

$V_0 = \frac{C}{\lambda_0} = 600 \text{ THz}$

(a) \[ \frac{\Delta V}{V} = \frac{\Delta \lambda}{\lambda} \quad \Rightarrow \quad \Delta V = 1.2 \times 10^{11} \text{ Hz} = 120 \text{ GHz} \]

\# of modes in volume $V$ is (within $\Delta V$):

(b) \[ N = \frac{8\pi V^2 \Delta V}{C^3} \times V \approx 8 \times 10^{10} \]

$\Delta V_{FSR} = \frac{C}{2d} = 750 \text{ MHz}$

\# of TEM$_{00}$ modes in 120 GHz interval is:

(c) \[ N_{TEM} = \frac{120 \times 10^9}{750 \times 10^6} \times 2 = 320 \]

Probability of spontaneous emission into one of TEM$_{00}$ is

(d) \[ \frac{320}{8 \times 10^{10}} \approx 4 \times 10^{-9} \quad [\text{small but possible}] \]
\[\sigma = \frac{A_2}{\gamma \Gamma n^2} \cdot g(V)\]

\[g(V) = \frac{1}{120 \cdot 6 + 2}\]

\[A = 10^7\quad \lambda_o = 0.5 \times 10^{-4}\text{ cm}\]

\[n = 1\]

\[\sigma(V_o) = \frac{10^7 \times (0.5 \times 10^{-4})^2}{8 \pi} \cdot \frac{1}{120 \times 10^9} \approx 8.3 \times 10^{-15}\text{ cm}^2\]