3.6. Consider a linear combination of two equal amplitude TEM_{n,p} modes given by:

\[ E = E_0 \left\{ (\text{TEM}_{1,0})a_x \pm j(\text{TEM}_{0,1})a_y \right\} \]

(a) Sketch the “dot” pattern or equal intensity contours for each component (i.e., \(a_x\) or \(a_y\)). Indicate the direction of the electric field.

(b) Sketch the pattern for the linear combination.

(c) Label the positions where the intensity is a maximum and a minimum. (This is sometimes referred to as the “donut mode” or TEM_{0,1}^*).

3.18. Is the cavity shown below stable? Demonstrate the logic of your answer by (a) constructing a unit cell starting at the flat mirror, (b) finding the ABCD matrix for that cell, and (c) applying the stability criteria. (d) What are the circumstances under which the quantity \([AD - BC]\) can be different from 1? Why is \(AD - BC\) always equal to 1 for a cavity?

(Ignore any astigmatism for problem 2).

3.21. A focused Gaussian beam reaches its minimum spot size \(w_0\) at \(z = 0\) where \(R = \infty\) and then propagates to a thin lens of focal length \(f\) located a distance \(d\) from \(z = 0\). If \(w_0\) is large, then the beam exiting the lens will be focused. If it is too small, then the lens merely reduces the far field spreading angle. Find the critical value of \(w_0\) such that the output beam is “collimated”; that is, \(R(z = d^+) = \infty\) also.

4. Using Eq. (5.2.8), write \(z_0^2\) in terms of the g-parameters (i.e. \(g_1\) and \(g_2\)). From this, derive the stability condition of the cavity in terms of \(g_1\) and \(g_2\). Compare this with the geometric optics results obtained earlier.