1. Verdeyen: Problem 9.41

9.41. The Erbium-doped fiber amplifier shown below is to be used as an amplifier in an optical communication link. Unfortunately, there are always some residual reflections $R_1$ and $R_2$, and, thus, if the gain were too high, then the amplifier will oscillate and the output will not be a faithful replica of the input. If the input signal were zero, then what is an expression for the maximum small signal single-pass gain $G_0$ that can be built into this amplifier. Evaluate for $R_1 = R_2 = 0.025$.

![Diagram of fiber amplifier](image)

2. Consider the following unidirectional ring laser. The following parameters are known for the homogeneously broadened gain medium:

- $\Delta \nu \approx 1 \times 10^{13} \text{ Hz}$, Gain cross section: $\sigma(\nu_0) = 2 \times 10^{-17} \text{ cm}^2$, $\lambda_0 = 1 \mu\text{m}$.
- Upper state lifetime $\approx 1 \mu\text{s}$, spot size (w) inside the gain medium $= 100 \mu\text{m}$ (Gaussian beam).
- $n($gain medium$) = 1.5$. Cavity parameters are given in the Fig. A.

![Diagram of ring laser](image)

(a) What is the threshold upper state population ($N_{2\text{th}}$)?
(b) What is the cw output power if this laser were to be pumped $\times 6$ above the threshold?
(c) Estimate the minimum excitation power required to sustain the output power in (b). Assume that the lower laser state (level 1 in Fig. B) is 3 eV above the ground state.
(d) If this laser were to be cw-modelocked, quantitatively describe (and graph) the temporal behavior of the output pulse-train. Assume the shortest possible pulse and ignore dispersion. Estimate the number of longitudinal modes that are oscillating. Estimate the peak output power if pumped at $\times 6$ above the threshold.
(e) If this laser were to be Q-switched, estimate the pulse width.
(f) What is the spontaneous lifetime of the gain medium?
3. Verdeyen: Problem 10.20

10.20. The problem is to model the following "Gedanken" experiment. We have an ideal laser medium (i.e., $\tau_1 = 0$ and hence $N_1 = 0$ always) with a stimulated emission coefficient of $10^{-15}$ cm$^2$, an upper state lifetime of 50 ns, a branching ratio of 0.6, a length of 15 cm and an area of 0.6 cm$^2$, and the wavelength is $\lambda = 5000$ Å. It is placed in the cavity shown below and the excitation rate of state 2, $R_2$, is varied from zero to three times the threshold for oscillation. There are two detectors shown: One is used to measure the spontaneous emission from the upper state and emerging from the side of the cavity, and the other is to measure the laser output along the axis of the cavity. Assume that the output of each detector is a current that is proportional to power.

Construct a graph showing the variation of both signals as a function of a normalized pumping rate $R_2/R_{\text{threshold}}$. Plot the total spontaneous and laser power in watts from this system. (The side-light detector would only capture a fraction of the total spontaneous amount, but that fraction would not be a variable.)
For oscillation we must have

\[ R_1 R_2 e^{2 \beta L_y} > 1 \text{ or } R_1 R_2 G_0 > 1 \]

\[ \Rightarrow G_0 > \frac{1}{\sqrt{R_1 R_2}} \]

So, in order to suppress oscillations we must have

\[ G_0 < \frac{1}{\sqrt{R_1 R_2}} \]

If \( R_1 = R_2 = 0.025 \), then \( G_0 < 40 = 16 \text{ dB} \).
1. \[ R = 0.999 \]

\[ R_1 = 0.999 \]

\[ R_2 = 0.95 \]

\[ T = 0.98 \]

\[ T_1 = 0.98 \]

\[ T_2 = 0.97 \]

\[ G_1 \approx 0 \]

\[ G_2 \approx 10 \]

\[ Y = 12 \]

---

(a) \[ \theta_{th} = \theta \left( N_2 - \frac{G_2}{G_1} N_1 \right) \]

\[ = \angle N^2 \]

Threshold condition is \[ R_1 R_2 T_1 T_2 R_3 e = 1 \]

\[ \Rightarrow \theta_{th} = \frac{1}{2} \ln \left( \frac{1}{R_1 R_2 R_3 T_1 T_2} \right) = 4.685 \, \text{m} \]

\[ \angle \theta_{th} = \frac{1}{2} \ln \left( \frac{0.999 \times 0.95}{0.98} \right) \]

\[ N_2 = \frac{\theta_{th}}{6} = \frac{4.685 \, \text{m}^2}{2 \times 10^{-21} \, \text{m}^2} = 2.34 \times 10^{21} \, \text{m}^3 = N_2^{th} \]

Threshold upper state population

(b) For a ray cavity:

\[ I_{out} = T_1 R_1 T_2 I_s \left( \frac{I_{pump}}{I_{pump,\text{threshold}}} - 1 \right) \]

\[ I_s = \frac{h \nu}{6 \pi \sigma_2} = \frac{h \left( \frac{C}{\sigma_2} \right)}{6 \pi \sigma_2} = 9.95 \times 10^{7} \, \text{w} \, \text{m}^2 \]
Spot size \( w = 100 \ \mu m \rightarrow r = 100 \ \mu m \)

\[
P = I_s A = I_s \pi r^2 = 9.95 \times 10^7 \times \pi \times (10^2) = 3.125 \ \text{W}
\]

\[
P_{\text{out}} = T_1 R T_2 R (\frac{I_{\text{pump}}}{I_{\text{pump \ threshold}}})
\]

\[
= 0.98 \times 0.999 \times 0.05 \times 3.125 \times (6-1) = 0.765 \ \text{W}
\]

(c) Minimum excitation power: \( P = R_2 E_2 \frac{\nu}{\text{volume}} \)

\[
R_2 = \frac{6}{R_2} = \frac{6}{N_2} = \frac{6}{g_2} = 6 \times \frac{234 \times 10^{21}}{10^{-6}} = 1.4 \times 10^{28} \ \text{m}^3
\]

\[
E_2 = 3 + \frac{\hbar c}{\lambda} = 3 + \frac{6.63 \times 10^{-34}}{1.6 \times 10^{-19}} \times 3 \times 10^8 = 3 + 1.24 = 4.24
\]

\[
P = 1.4 \times 10 \times 4.24 \times 1.6 \times 10^9 \times \pi \times (100 \times 10^{-6})^2 \times 0.02 = 5.96 \ \text{W}
\]

\[
\boxed{P = 5.96 \ \text{W}}
\]
\[
\frac{1}{\Delta \tau_{\text{PSR}}} = \frac{\Delta \tau_{\text{RT}}}{C} = \frac{(0.27 + 0.02 \times 1.5)}{C} = 10 \times 10^{-10} = 1 \text{ ns}
\]

\[\Delta t_{p} \approx \frac{1}{\Delta \tau_{\text{PSR}}} = 10^{-13} \text{ sec.} = 10 \text{ fs.}\]

\[\text{# of modes that are oscillating} = \frac{\Delta \tau_{\text{PSR}}}{\Delta \tau_{\text{PSR}}} = 10^4 \]

\[\frac{P_{\text{peak}}}{P_{CW}} \rightarrow P_{\text{peak}} = 1 \times 10^4 \times 0.765 = 7.65 \times 10^3 \text{ W}\]

\[\text{Peak intensity: } I_{\text{peak}} = \frac{P_{\text{peak}}}{A} = \frac{7.65 \times 10^3}{\pi \times (100 \times 10^{-6})^2} = 2.48 \times 10^4 \text{ W/m}^2\]

\[(e) \quad \Delta f_{\text{eq.s.}} = \frac{1}{\Delta f_{\text{photon}}} \quad \Delta f_{\text{photon}} = \frac{\Delta \tau_{\text{RT}}}{1 - S}\]

\[\Delta f_{\text{photon}} = \frac{10^{-9} \text{ sec.}}{1 - (0.999)^2 (0.98)^2 (0.95)} = 1.11 \times 10^8\]

\[\Rightarrow \Delta f_{\text{eq.s.}} = \frac{1}{1.11 \times 10^8} = 9 \times 10^7 \text{ Hz.}\]
\( \psi = \frac{A_{21} \lambda^2}{8\pi n^2} g(\nu) \)

Assuming \( g(\nu) \) is in central frequency, \( g(\nu) \approx \frac{1}{\Delta \nu} \)

\[ \Rightarrow A_{21} = \frac{\psi 8\pi n^2 \lambda^2}{\Delta \nu} = \frac{1.13 \times 10^{-6}}{(10^{-6})^2} = 1.13 \times 10^6 \text{ /sec.} \]

\[ f_{21} = \frac{1}{A_{21}} = 0.88 \text{ ms.} \]
(10.20) Verdeyen 3rd ed.

\[ 0.6 \text{ cm}^2 \]

Active medium

\[ R_1 = 0.95 \]
\[ T_1 = 0.0 \]

\[ R_2 = 0.7 \]
\[ T_2 = 0.25 \]

\[ g = 15 \text{ cm} \]

\( f_1 = 0 \) (i.e. \( N_1 = 0 \) always).

\( s = 10^{-15} \text{ cm}^2 \); \( \tau = 50 \text{ ns} \); branching ratio \( \eta = 0.6 \) = \( \text{rate of decay from } 2 \rightarrow 1 \) / \( \text{sum of all decay rates of } 2 \).

\( \lambda = 5000 \text{ Å} \); \( R_2 \) may vary from zero to three times the threshold for oscillation.

\[ \begin{array}{c}
2 \\
\uparrow \\
R_2 \\
0 \\
\downarrow \\
N_1 = 0
\end{array} \]

\[ 5000 \text{ Å} \]

Rate of \( N_2 \) for level 2. before threshold

\[ \frac{dN_2}{dt} = R_2 - \frac{N_2\Phi}{\tau} - \frac{N_2(1-\Phi)}{\tau} = 0 \]

spontaneous emission from level 2 \( \rightarrow 1 \)  spontaneous emission from level 2 \( \rightarrow 0 \)

\[ N_2 = \frac{1}{\Phi} R_2 \] (and go a result \( N_2^{th} = R_2^{th} \) \( \Phi \))
First, let's calculate what $N_2^{th}$ & $R_2^{th}$ are.

Threshold condition, $R_1 R_2 T a^2 T_b \geq 4 \ln 2 \Rightarrow I = 1$

$$\Rightarrow \kappa_{th} = \frac{1}{2 a} \ln \frac{1}{R_1 R_2 T a^2 T_b} = 1.84 \text{ m}^{-1}$$

$$\Rightarrow \kappa_{th} = 2 \kappa_{th} = \frac{N_2^{th}}{A} = 1.84 \times 10^{-19} = 1.84 \times 10^{-19} \text{ m}^3$$

$$R_2^{th} = \frac{N_2^{th}}{J} = \frac{1.84 \times 10^{-19}}{3.68 \times 10^{-26}} \approx 3.68 \times 10^{-3} \text{ sec.}$$

Spontaneus emission power before threshold is obtained by

$$P_{sp} = \frac{h \kappa_{th} N_2 V}{J} = \frac{h \kappa_{th} \kappa_{th} R_2^2}{J} = \frac{h \kappa_{th} R_2^2}{J} R_2 \times \frac{R_2}{R_2^{th}}$$

$$= \frac{h \kappa_{th} R_2^{th}}{J} = \frac{h \kappa_{th}}{J} \times 6.63 \times 10^{-34} \times 3 \times 10^8 \times 0.6 \times (15 \times 0.6 \times 10^{-6})$$

$$\times 3.68 \times 10^{-26} \frac{R_2}{R_2^{th}}$$

$$\Rightarrow P_{sp} = 790 \frac{R_2}{R_2^{th}} \text{ W}$$
Stimulated emission must be considered after threshold,

\[ \frac{dN_2}{dt} = R_2 - \frac{N_2 I^+}{g} - \frac{N_2 (1 - \eta)}{g} - \frac{6I^+}{h \nu} N_2 = 0 \]

I^+ is the stimulated emission intensity inside the active medium.

\[ \begin{pmatrix} I^+ \\ \hline \rightarrow \\ \hline \leftarrow \end{pmatrix} \]

approximately, the intensity inside active medium is

\[ I = I^+ + I^- \approx 2I^+ = 2 \frac{I_0}{2} \left( \frac{R_2}{R_2^{th}} - 1 \right) \]

\[ \Rightarrow R_2 = N_2 \left( \frac{1}{g} + \frac{g}{h \nu} I_5 \left( \frac{R_2}{R_2^{th}} - 1 \right) \right) \quad \text{; but } I_5 = \frac{h \nu}{6g} \]

\[ \Rightarrow R_2 = N_2 \left( \frac{1}{g} + \frac{1}{g} \left( \frac{R_2}{R_2^{th}} - 1 \right) \right) = N_2 \frac{1}{g} \frac{R_2}{R_2^{th}} \]

\[ \Rightarrow N_2 = g R_2^{th} \]\n
(which means that \( N_2 = N_2^{th} \) after lasing starts)

\[ \begin{aligned} \text{after threshold} & \quad \leftarrow (a) \quad P_{sp} = \frac{h \nu \eta \Phi}{g} N_2 V = \frac{h \nu \eta \Phi V g R_2^{th}}{g} = h \nu \Phi V R_2^{th} \quad \text{const.} \\
\end{aligned} \]

\[ \begin{aligned} & \quad \text{(a)} \quad P_{sp} = 6.63 \times 10^{-34} \times \frac{3 \times 10^{8}}{5 \times 10^{-7}} \times 0.6 \times (15 \times 0.6 \times 10^{10}) \times 3.68 \times 10 = 7.9 \text{ W} \\
& \quad \text{(a)} \quad P_{sp}' = 790 \text{ W} \end{aligned} \]
The stimulated emission power (or the laser output power) is

\[ P_{\text{out}} = \frac{P_{3T2}}{2} \left( \frac{R_2}{R_{2\text{th}}} - 1 \right) \]

\[ I_5 = \frac{P_2}{\sigma J} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{5 \times 10^{-7} \times 10^{-19} \times 50 \times 10^{-9}} = 7.95 \times 10^7 \frac{\text{W}}{\text{m}^2} \]

\[ P_5 = I_5 \times A = 7.95 \times 10^7 \times (0.6 \times 10^{-4}) = 4.7 \times 10^7 \text{ W} \]

\[ P_{\text{out}} = \frac{4.7 \times 10^7 \times 0.25}{2} \left( \frac{R_2}{R_{2\text{th}}} - 1 \right) \]

\[ \Rightarrow P_{\text{out}} = 596 \left( \frac{R_2}{R_{2\text{th}}} - 1 \right) \text{ W} \]

\[ P \text{ (W)} \]

The inversion is clamped at threshold.

\[ \frac{R_2}{R_{2\text{th}}} \]