Laser Physics I (PHYC/ECE 464)
FALL 2020

OPTICAL SCIENCE & ENGINEERING
University of New Mexico

Final Exam, Closed Book, Closed Notes

Time: 3:00 – 5:00 pm

Total= 100 points

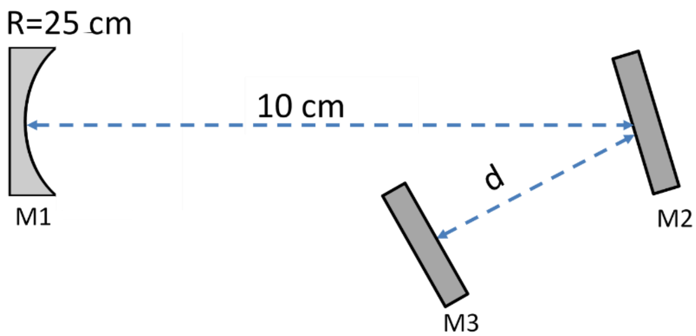
Please staple these pages with your solutions.

Have a Safe and Happy Thanksgiving!

Instructor: M. Sheik-Bahae

1. (20 points) Consider a simple standing-wave cavity constructed from a concave and two flat reflectors as shown below.

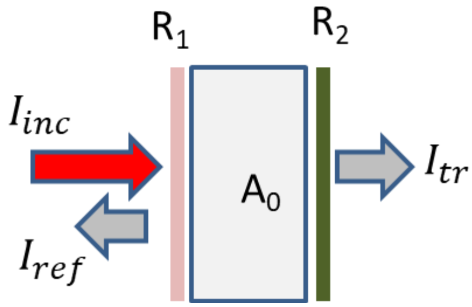
(You do not need to use the ABCD matrices to solve this problem)



a) Give an expression (as a function of d) for beam radius w at mirror M2 for a wavelength λ_0.

b) What find the range of d for which the cavity is stable.

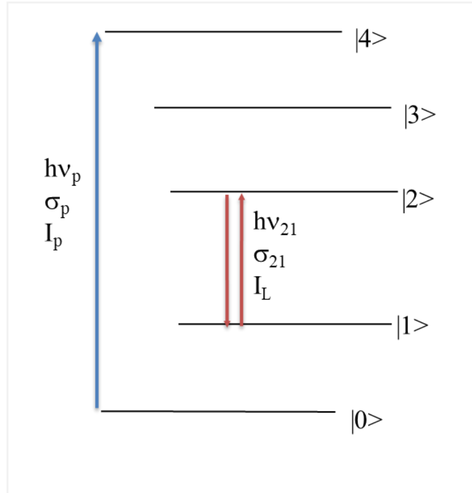
2. *(20 points)* A Fabry-Perot (FP) cavity formed by two mirrors having reflectivity R_1 and R_2 is filled with an absorber having an absorbance $A_0 (= e^{-\alpha d} < 1)$.



Assuming on-resonance ($\theta = kd = m\pi$), can you find values for **R_1 and R_2** (anywhere between 0 to 100%) for which all the incident power is absorbed inside the FP? (i.e. $I_{tr} = I_{ref} = 0$) .

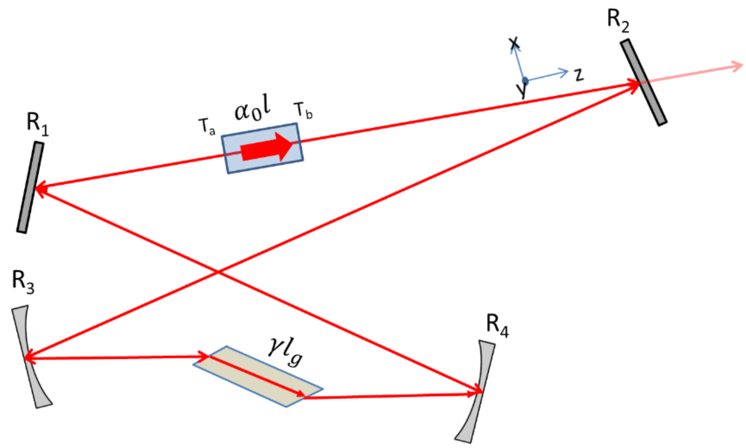
*This simple yet interesting condition has been called "**perfect coherent absorber**" or "**critical coupling**".*

3. Rate Equations: *(20 points*) Write down the rate equations for the following 5-level system where optical pumping is from the ground state to level 4, and stimulated absorption/emission is between levels 1 and 2.



The known paramers (in addtion to those shown in the figure) are each level liftime τ_j (j=1,2,3,4), branching rations ϕ_{ji} (i<j), and the total atomic density (N_t). Assume all degnerency factors are unity.

4. *(20 points*) Consider the unidirectional ring cavity shown below. The gain medium is an ideal Brewster-cut crystal . There is an absorber in the cavity with absorbence $A_0 = \alpha_0 l$ and imperfet surface reflectivities.



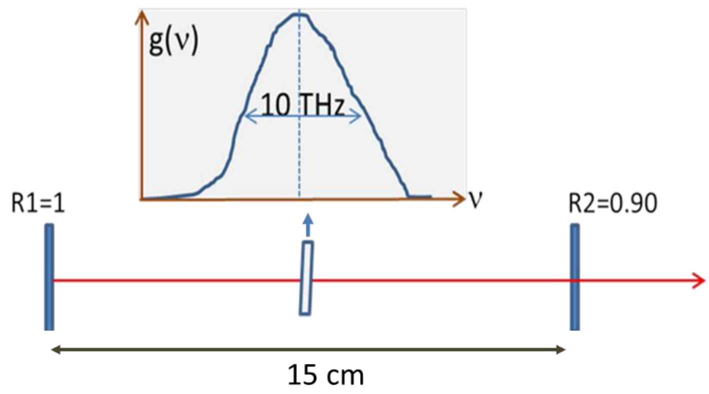
(a) What is the prefered laser polarization (x,y or z) , and why?

(b) What is the survival factor of the passive cavity?.

(c) What is the threshold gain γ_{th}?

(d) Pumped N-times above threshold ($\frac{\gamma_0}{\gamma_{th}} = N$), what is the output intensity (through mirror 2)? You may assume high -Q cavity.

5. Consider the laser system shown: *(20 points)*



a. If Q-switched, approximately sketch the pulse shape assuming two cases of (i) one and (ii) two longitudinal modes? Be semi-quantitative on your time-axis. {e.g. approximate pulsewidth)

b. If cw- modelocked, approximately sketch the pulse train assuming the shortest pulse. How many longitudinal modes will oscillate? Be semi-quantitative on your time-axis

Final Formula Sheet
PHYS/ECE 464 (Laser Physics I)- University of New Mexico (2020)

Hermite-Gaussian Beams

$$\frac{E(x,y,z)}{E_0} = H_m\left(\frac{\sqrt{2}x}{w(z)}\right)H_p\left(\frac{\sqrt{2}y}{w(z)}\right)\frac{w_0}{w(z)}\exp\left(-i\frac{kr^2}{2q(z)}\right)\times\exp\left(-i\left[kz-(1+m+p)\tan^{-1}(z/z0)\right]\right)$$

$$\frac{1}{q(z)} = \frac{1}{R(z)} - i\frac{\lambda_0}{\pi n w^2(z)}, \quad w^2(z) = w_0^2\left(1+\frac{z^2}{z_0^2}\right), \quad R(z) = z\left(1+\frac{z_0^2}{z^2}\right), \quad z_0 = \frac{\pi n w_0^2}{\lambda_0} \quad k = n\frac{\omega}{c} = \frac{2\pi n}{\lambda_0}$$

Irradiance: $I = \langle S \rangle = \frac{nc\varepsilon_0}{2}E_0{}^2$

Snell's Law $n_i\sin(\theta_i) = n_t\sin(\theta_t)$

Fresnel's reflectivities:

$$r_{\parallel} = \frac{n_t\cos(\theta_i) - n_i\cos(\theta_t)}{n_t\cos(\theta_i) + n_i\cos(\theta_t)} = \frac{\tan(\theta_i-\theta_t)}{\tan(\theta_i+\theta_t)}$$

$$r_{\perp} = -\frac{n_i\cos(\theta_i) - n_t\cos(\theta_t)}{n_i\cos(\theta_i) + n_t\cos(\theta_t)} = -\frac{\sin(\theta_i-\theta_t)}{\sin(\theta_i+\theta_t)}$$

Intensity (Power) reflectivity: $R=|r|^2$

Brewster angle (from 1 to 2): $\theta_B = \tan^{-1}(n_2/n_1)$

Critical angle (from 1 to 2): $\theta_c = \sin^{-1}(n_1/n_2)$

$n \rightarrow \tilde{n} = n + i\kappa$ *in n complex*

Lens Transformation of a Gaussian beam: $\frac{1}{R_{out}} = \frac{1}{R_{in}} - \frac{1}{f}$	$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ Lens-makers' formula:

Fabry-Perot Transmission and Reflection (with gain G_0 >1 or loss A_0<1)

$T(\theta,G_0) = \frac{G_0(1-R_1)(1-R_2)}{\left(1-G_0\sqrt{R_1R_2}\right)^2 + 4G_0\sqrt{R_1R_2}\sin^2(\theta)}$	*Finesse* $F = \frac{\pi\sqrt[4]{R_1R_2}}{1-\sqrt{R_1R_2}} = \frac{\Delta\nu_{FSR}}{\Delta\nu_{1/2}}$
$R(\theta,G_0) = \frac{\left(\sqrt{R_1} - G_0\sqrt{R_2}\right)^2 + 4G_0\sqrt{R_1R_2}\sin^2(\theta)}{\left(1-G_0\sqrt{R_1R_2}\right)^2 + 4G_0\sqrt{R_1R_2}\sin^2(\theta)}$	*Free Spectral Range:* $\Delta\nu_{FSR} = \frac{c}{2nd} = \frac{1}{\tau_{RT}}$ *Photon Lifetime:* $\tau_p = \frac{\tau_{RT}}{1-R_1R_2} \approx \frac{1}{2\pi\Delta\nu_{1/2}}$
$2\Delta\theta_{1/2} = \frac{1-G_0\sqrt{R_1R_2}}{G_0^{1/2}\sqrt[4]{R_1R_2}}$ $\theta = kd = \frac{\omega nd}{c}$ *For absorption replace G_0 with A_0(<1)*	*General Resonance Condition:* *roundtrip phase change= q2π*

ABCD Matrices $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$ AD-BC=1 $\begin{pmatrix} r_2 \\ r_2' \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} r_1 \\ r_1' \end{pmatrix}$

Free space of length d $\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$	Dielectric interface (from n_1 to n_2) $\begin{pmatrix} 1 & 0 \\ 0 & n_1/n_2 \end{pmatrix}$	ABCD rule for Gaussian Beams $q_2 = \frac{Aq_1+B}{Cq_1+D}$ *where*	$q(z) = z + iz_0$ *or* $\frac{1}{q(z)} = \frac{1}{R(z)} - i\frac{\lambda_0}{\pi n w(z)^2}$
medium of length d and index n_2=n immersed in vacuum (n_1=1). $\begin{pmatrix} 1 & d/n \\ 0 & 1 \end{pmatrix}$	Thin lens of focal length f $\begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$	Gaussian pulse propagation (broadening) in dispersive media $\tau_p^2(z) = \tau_{p0}^2\left(1+\frac{z^2}{\ell_0^2}\right)$ where $\ell_0 = \frac{\tau_{p0}^2}{2\lvert\beta_2\rvert}$ and group velocity dispersion (GVD) $\beta_2 = \frac{\lambda^3}{2\pi c^2}\frac{d^2n}{d\lambda^2} = \frac{\lambda^2}{2\pi c}D$	
Mirror with radius of curvature R $\begin{pmatrix} 1 & 0 \\ -2/R & 1 \end{pmatrix}$	Spherical dielectric interface $\begin{pmatrix} 1 & 0 \\ (1-n_1/n_2)/R & n_1/n_2 \end{pmatrix}$	Photon Density (Photon Number per Volume) $\frac{N_p}{V} = \frac{I}{h\nu c/n_g}$	

Gain in a two-level system: $\gamma(\nu) = \sigma(\nu)\left[N_2 - \frac{g_2}{g_1}N_1\right]$ Gain cross section: $\sigma(\nu) = A_{21}\frac{\lambda^2}{8\pi n^2}g(\nu)$

Lineshape Normalization: $\int g(\nu)d\nu = 1$ Beer's Law: $\frac{1}{I}\frac{dI}{dz} = -\alpha(I) + \gamma(I)$

Gain or absorption saturation in a homogenously-broadened system:

$\gamma(I) = \frac{\gamma_0}{1 + I/I_s}$ or $\alpha(I) = \frac{\alpha_0}{1 + I/I_s}$ $I_s(\nu) = \frac{h\nu}{\sigma(\nu)\tau_2}$

Einstein's relation: $\frac{A_{21}}{B_{21}} = \frac{8\pi n^3 h\nu^3}{c^3}$ $g_2 B_{21} = g_1 B_{12}$ $\frac{N_2}{N_1} = \frac{g_2}{g_1}e^{-(E_2-E_1)/kT}$

Lorentzian line shape:	Doppler broadened line shape
$g(\nu) = \frac{\Delta\nu_h/2\pi}{(\nu-\nu_0)^2 + (\Delta\nu_h/2)^2}$	$g(\nu) = \left(\frac{4\ln 2}{\pi}\right)^{1/2}\frac{1}{\Delta\nu_D}\exp\left[(-4\ln 2)\left(\frac{\nu-\nu_0}{\Delta\nu_D}\right)^2\right]$ with $\Delta\nu_D = \left(\frac{8kT\ln 2}{Mc^2}\right)^{1/2}\nu_0$

$\frac{dN_p}{dt} = \frac{G^2S - 1}{\tau_{RT}}N_p + N_2 c\sigma$ (Photon number dynamics due to *stimulated* and *spontaneous* emission)

S (survival factor)=R_1R_2 (only) for a simple two mirror linear cavity, G^2=roundtrip gain =e^{2g}, with $g=\gamma L_g$ (integrated gain). Threshold condition: $SG^2=1$ (linear cavity), SG=1 (ring cavity)

Schawlow-Townes limit for laser linewidth: $\Delta\nu_{osc} \approx 2\pi\frac{h\nu}{P_{out}}(\Delta\nu_{1/2})^2$

At steady-state: $\boldsymbol{\gamma = \gamma_{th} = \frac{\gamma_0}{1+I/I_s}}$ (for homogenously broadened)

Inside the gain medium: $I \approx I^+ + I^- \approx 2I^+$ for a high-Q linear (standing-wave) or bidirectional ring cavity, $I \approx I^+$ for a unidirectional ring cavity.

$\mathbf{I_{out}=T_a..T_2 I^+}$ (T_2 is the output coupling transmission and T_a… are the transmission of other optical surfaces in the path).

Optimum output coupling: $\mathbf{T_2^{opt} = -L_i + (g_0 L_i)^{1/2}}$ where L_i accounts for roundtrip internal (useless) losses , $g_0=\gamma_0 l_g$ is the unsaturated (small signal) integrated gain. l_g is the length of the gain medium.

Q-Switching and Gain-Switching: $\Delta t_p \approx \tau_p$ (cavity photon lifetime)

Modelocking: Repetition Rate $=1/T_{rt}=2Ln_g/c$ (linear cavity), Pulsewidth: $\Delta t_p \approx > 1/\Delta\nu$

Threshold current density in a diode laser: $J_{th} = eN_{eh}^{th}d/\tau_r$

Physical Constants

$c \sim 3\times10^8$ m·s^{-1}	$h=6.63\times10^{-34}$ J·s	$e=1.6\times10^{-19}$ C
$k_B=1.380\times10^{-23}$ J·K^{-1}	$\varepsilon_0=8.85\times10^{-12}$ F/m	$m_e=9.1\times10^{-31}$ kg

1G = 10^{-4} T , 1 eV = 1.602×10^{-19} J, 1 dyne = 10^{-5} N, 1 erg = 10^{-7} J