Laser Physics I (PHYC/ECE 464)
FALL 2019

Final Exam, Closed Book, Closed Notes
No computers, No cell-phones. Calculators will be provided
Dec. 9, 2019, Time: 5:30 – 7:30 pm

NAME ....
last *first*

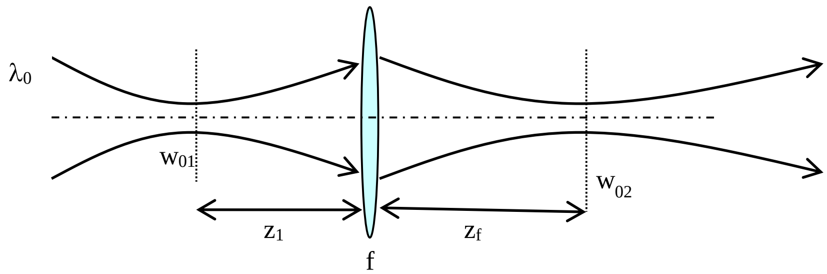
Score

Total= 100

- ***Write your name on each sheet.***
- ***Please staple and return these pages with your exam.***

Instructor: M. Sheik-Bahae

1. ***(30 points)*** A laser beam with wavelength $\boldsymbol{\lambda_0}$ is incident from left on a lens with focal length **f** placed at a distance $\mathbf{z_1}$ after its minimum waist, as shown below.



(a) (*15 pts*.) What is the **q-**parameter, beam radius **w** and radius of curvature **R** at the following locations:

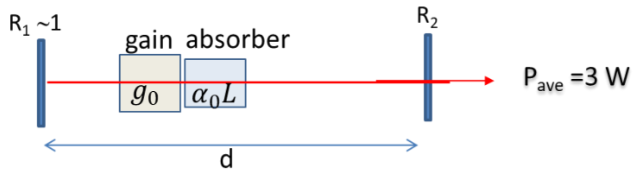
a.1 Just before the lens?

a.2 Right after the lens?

a.3 At an arbitrary distance z_2 after the lens?

(b) (*15 pts.)* Show how to determine the location (z_f) and the value (w_{02}) of the new focus in terms of the given parameters.

2. ***(40 points)*** Consider the laser system below with an average CW output power of **3 Watts** with an optimized output coupling (T_2^{opt}) when pumped at an integrated small-signal gain of **g_0=0.25**. The only internal loss is from an intracavity absorber with **$\alpha_0 L$=0.02**. All other intracavity surfaces are considered lossless (AR coated). The beam radius inside the cavity is **w~80 μm** inside the gain and absorber media.



(a) What is the output coupler reflectivity **R_2**?

(b) What are g_{th} and γ_0/γ_{th} ?

(c) What is the *total intensity* inside the gain medium and the gain saturation intensity? (Assume and justify a high-Q cavity, and assume homogeneously broadened gain)

(d) Estimate the total power absorbed in the absorber.

This laser may be CW mode-locked to generate a train of pulses with **100 MHz** repetition rate and a peak pulse power of P_{peak}~ **300 kW**.

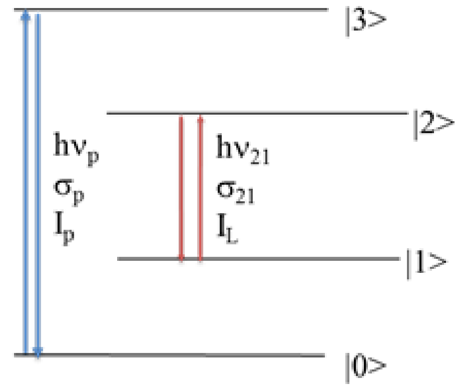
(e) Estimate the pulsewidth Δt_p ?

(f) What is the effective cavity length d (assume n_g=1) ?

(g) What is the saturation intensity of the absorber knowing that it saturates to **90%** to its unsaturated value when the laser is mode-locked.

(h) What *lifetime characteristics* render the gain and the saturable absorber suitable for modelocking?

3. ***(30 points)***



(a) *(10 pts.)* Write the rate equations for this 4-level system assuming optical pumping from levels $0 \rightarrow 3$ and laser action from levels $2 \rightarrow 1$. Lifetimes τ_3, τ_2, τ_1, and branching rations ϕ_{32}, ϕ_{31}, ϕ_{30}, ϕ_{21}, ϕ_{20} , and degeneracy factors g_0, g_1, g_2, g_3 are known.

(b) *(10 pts.)* The above system is to be used as a laser amplifier. Assume $\nu_p=2\nu_{12}$, $\phi_{31}\sim\phi_{30}\sim\phi_{20}\sim0$, $\phi_{32}=0.8$, $\tau_1\sim0$. If 15W of pump power is absorbed, what is the maximum power that can be possibly extracted from this amplifier.

(c) *(10 pts.*) If λ_{12}**=1 μm,** τ_2**=1 μs**, estimate the ***total population*** in level 2 for part **3-(b)**

Formula Sheet

PHYC/ECE 464 (Laser Physics I)- University of New Mexico (2019)

Instructor: Mansoor Sheik-Bahae

Hermite-Gaussian Beams

$$\frac{E(x,y,z)}{E_0} = H_m\left(\frac{\sqrt{2}x}{w(z)}\right) H_p\left(\frac{\sqrt{2}y}{w(z)}\right) \frac{w_0}{w(z)} \exp\left(-i\frac{kr^2}{2q(z)}\right) \times \exp\left(-i\left[kz-(1+m+p)\tan^{-1}(z/z0)\right]\right)$$

$$\frac{1}{q(z)} = \frac{1}{R(z)} - i\frac{\lambda_0}{\pi n w^2(z)}, \quad w^2(z) = w_0^2\left(1+\frac{z^2}{z_0^2}\right), \quad R(z) = z\left(1+\frac{z_0^2}{z^2}\right), \quad z_0 = \frac{\pi n w_0^2}{\lambda_0} \quad k = n\frac{\omega}{c} = \frac{2\pi n}{\lambda_0}$$

Irradiance: $I = \langle S \rangle = \frac{nc\varepsilon_0}{2} E_0^{\,2}$

Fresnel's reflectivities:

$$r_{\parallel} = \frac{n_t\cos(\theta_i) - n_i\cos(\theta_t)}{n_t\cos(\theta_i) + n_i\cos(\theta_t)} = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}$$

$$r_{\perp} = -\frac{n_i\cos(\theta_i) - n_t\cos(\theta_t)}{n_i\cos(\theta_i) + n_t\cos(\theta_t)} = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}$$

Snell's Law $n_i\sin(\theta_i) = n_t\sin(\theta_t)$

Intensity (Power) reflectivity: $R=|r|^2$

Brewster angle (from 1 to 2): $\theta_B = \tan^{-1}(n_2/n_1)$

Critical angle (from 1 to 2): $\theta_c = \sin^{-1}(n_1/n_2)$

$n \rightarrow \tilde{n} = n + i\kappa$ *in n complex*

Lens Transformation of a Gaussian beam: $\frac{1}{R_{out}} = \frac{1}{R_{in}} - \frac{1}{f}$	$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ Lens-makers' formula:

Fabry-Perot Transmission and Reflection (with gain or loss)

$$T(\theta, G_0) = \frac{G_0(1-R_1)(1-R_2)}{\left(1-G_0\sqrt{R_1R_2}\right)^2 + 4G_0\sqrt{R_1R_2}\sin^2(\theta)}$$

$$R(\theta, G_0) = \frac{\left(\sqrt{R_1} - G_0\sqrt{R_2}\right)^2 + 4G_0\sqrt{R_1R_2}\sin^2(\theta)}{\left(1-G_0\sqrt{R_1R_2}\right)^2 + 4G_0\sqrt{R_1R_2}\sin^2(\theta)}$$

$$2\Delta\theta_{1/2} = \frac{1-G_0\sqrt{R_1R_2}}{G_0^{1/2}\sqrt[4]{R_1R_2}}$$

$$\theta = kd = \frac{\omega n d}{c}$$

Finesse $F = \frac{\pi\sqrt[4]{R_1R_2}}{1-\sqrt{R_1R_2}} = \frac{\Delta\nu_{FSR}}{\Delta\nu_{1/2}}$

Free Spectral Range: $\Delta\nu_{FSR} = \frac{c}{2nd} = \frac{1}{\tau_{RT}}$

Photon Lifetime:

$$\tau_p = \frac{\tau_{RT}}{1 - R_1R_2} \approx \frac{1}{2\pi\Delta\nu_{1/2}}$$

General Resonance Condition:
roundtrip phase change= $q2\pi$

ABCD Matrices $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$ AD-BC=1 $\begin{pmatrix} r_2 \\ r_2' \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} r_1 \\ r_1' \end{pmatrix}$

Free space of length d $\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$	Dielectric interface (from n_1 to n_2) $\begin{pmatrix} 1 & 0 \\ 0 & n_1/n_2 \end{pmatrix}$	ABCD rule for Gaussian Beams $q_2 = \frac{Aq_1 + B}{Cq_1 + D}$ *where*	$q(z) = z + iz_0$ *or* $\frac{1}{q(z)} = \frac{1}{R(z)} - i\frac{\lambda_0}{\pi n w(z)^2}$
medium of length d and index n_2=n immersed in vacuum (n_1=1). $\begin{pmatrix} 1 & d/n \\ 0 & 1 \end{pmatrix}$	Thin lens of focal length f $\begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$	Gaussian pulse propagation (broadening) in dispersive media $\tau_p^2(z) = \tau_{p0}^2\left(1+\frac{z^2}{\ell_0^2}\right)$ where $\ell_0 = \frac{\tau_{p0}^2}{2\lvert\beta_2\rvert}$ and group velocity dispersion (GVD) $\beta_2 = \frac{\lambda^3}{2\pi c^2}\frac{d^2n}{d\lambda^2} = \frac{\lambda^2}{2\pi c}D$	
Mirror with radius of curvature R $\begin{pmatrix} 1 & 0 \\ -2/R & 1 \end{pmatrix}$	Spherical dielectric interface $\begin{pmatrix} 1 & 0 \\ (1-n_1/n_2)/R & n_1/n_2 \end{pmatrix}$	Photon Density (Photon Number per Volume) $\frac{N_p}{V} = \frac{I}{h\nu c/n_g}$	

Gain in a two-level system: $\gamma(\nu) = \sigma(\nu)\left[N_2 - \frac{g_2}{g_1}N_1\right]$ Gain cross section: $\sigma(\nu) = A_{21}\frac{\lambda^2}{8\pi n^2}g(\nu)$

Lineshape Normalization: $\int g(\nu)d\nu = 1$ Beer's Law: $\frac{1}{I}\frac{dI}{dz} = -\alpha(I) + \gamma(I)$

Gain or absorption saturation in a homogenously-broadened system:

$\gamma(I) = \frac{\gamma_0}{1+I/I_s}$ or $\alpha(I) = \frac{\alpha_0}{1+I/I_s}$ $I_s(\nu) = \frac{h\nu}{\sigma(\nu)\tau_2}$

Einstein's relation: $\frac{A_{21}}{B_{21}} = \frac{8\pi n^3 h\nu^3}{c^3}$ $g_2 B_{21} = g_1 B_{12}$ $\frac{N_2}{N_1} = \frac{g_2}{g_1}e^{-(E_2-E_1)/kT}$

Lorentzian line shape: $g(\nu) = \frac{\Delta\nu_h/2\pi}{(\nu-\nu_0)^2 + (\Delta\nu_h/2)^2}$	*Doppler broadened line shape* $g(\nu) = \left(\frac{4\ln 2}{\pi}\right)^{1/2}\frac{1}{\Delta\nu_D}\exp\left[(-4\ln 2)\left(\frac{\nu-\nu_0}{\Delta\nu_D}\right)^2\right]$ with $\Delta\nu_D = \left(\frac{8kT\ln 2}{Mc^2}\right)^{1/2}\nu_0$

$\frac{dN_p}{dt} = \frac{G^2S-1}{\tau_{RT}}N_p + N_2 c\sigma$ (Photon number dynamics due to *stimulated* and *spontaneous* emission)

S (survival factor)=R_1R_2 for a simple two mirror linear cavity, G^2=roundtrip gain ($\exp(2\gamma L_g)$)
Threshold condition: $SG^2=1$ (linear cavity), $SG=1$ (ring cavity)

Schawlow-Townes limit for laser linewidth: $\Delta\nu_{osc} \approx 2\pi\frac{h\nu}{P_{out}}(\Delta\nu_{1/2})^2$

At steady-state: $\gamma = \gamma_{th} = \frac{\gamma_0}{1+I/I_s}$ (for homogenously broadened)

Inside the gain medium: $I \approx I^+ + I^- \approx 2I^+$ for a high-Q linear (standing-wave) or bidirectional ring cavity, $I \approx I^+$ for a unidirectional ring cavity.
$I_{out} = T_a..T_2 I^+$ (T_2 is the output coupling transmission and T_a… are the transmission of other optical surfaces in the path).

Optimum output coupling: $T_2^{opt} = -L_i + (g_0 L_i)^{1/2}$ where L_i accounts for roundtrip internal (useless) losses, $g_0 = \gamma_0 l_g$ is the unsaturated (small signal) integrated gain. l_g is the length of the gain medium.

Q-Switching and Gain-Switching: $\Delta t_p \approx \tau_p$ (cavity photon lifetime)

Mode-locking: Repetition Rate $=1/T_{rt} = 2Ln_g/c$ (linear cavity), Pulsewidth: $\Delta t_p \approx > 1/\Delta\nu$

Threshold current density in a diode laser: $J_{th} = eN_{eh}^{th}d/\tau_r$

Physical Constants

$c=2.998 \times 10^8$ m·s^{-1}	$h=6.63 \times 10^{-34}$ J·s	$e=1.6 \times 10^{-19}$ C
$k_B=1.380 \times 10^{-23}$ J·K^{-1}	$\varepsilon_0=8.85\times10^{-12}$ F/m	$m_e=9.1 \times 10^{-31}$ kg

1G = 10^{-4} T , 1 eV = 1.602×10^{-19} J, 1 dyne = 10^{-5} N, 1 erg = 10^{-7} J