Laser Physics I (PHYC/ECE 464)
FALL 2012

Midterm Exam, Closed Book, Closed Notes

Time: 5:30 – 6:45 pm

NAME ....
last *first*

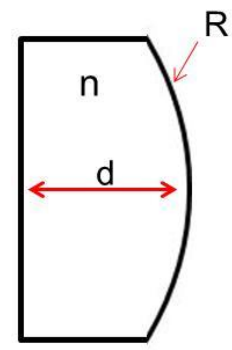
Score

Total= 100 points

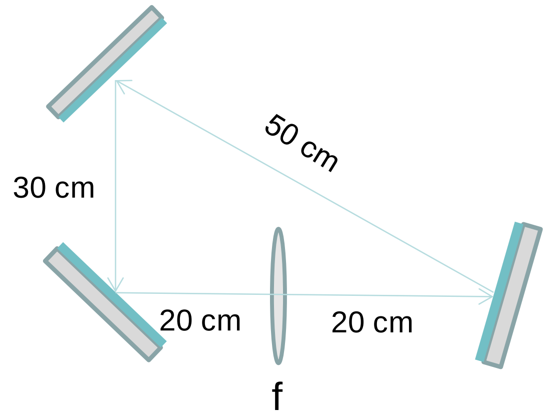
Please staple and return these pages with your exam.

Instructor: M. Sheik-Bahae

1. (a) Derive the ray matrix for the "thick" plano-convex lens shown below: *(20 points)*



2. ***(30 points)*** Consider the ring cavity (shown below) consisting of one lens and 3 flat mirrors.



(a) Where is the position of minimum beam waist w_0? Explain and mark the position on cavity diagram. (*no calculations is necessary for this part*) *(5pts.)*

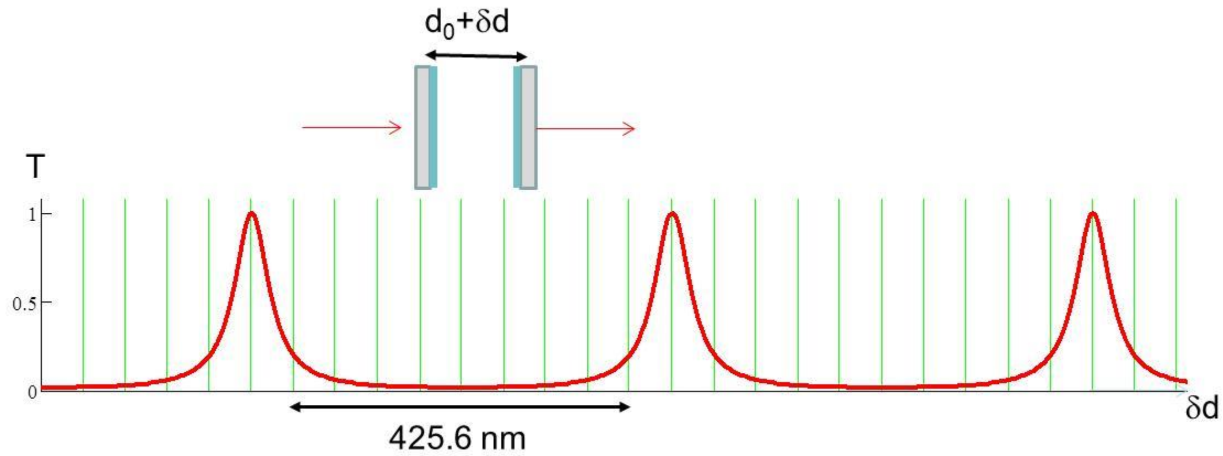
(b) For a proper starting point of your choice, identify a unit cell and write down the matrix product for a roundtrip. *(7pts.)*

(c) Give the range of focal lengths f for which this cavity is stable. *(8pts.)*

(d) Obtain the Rayleigh range Z_0 as a function of f. *(10 pts.)*

3. ***(25 points)***

Drawn to scale in the graph below is the power transmission of a scanning Fabri-Perot as the distance is increased from its intial 1cm to 1cm+1.44μm. The source is a single wavelength laser at wavelength λ_0.



a) What is λ_0? *(5pts.)*

b) What is $\Delta\nu_{1/2}$ (in MHz)? *(7 pts.)*

c) What is the finesse? *(6 pts.)*

d) What is the photon lifetime? *(7 pts.)*

4. ***(25 points)*** Consider a pressure-broadened gaseous two-level medium with the following property:

- *Spontaneous emission lifetime:* τ_{sp}*=1* μs
- *Homogeneous linewidth* $\Delta\nu_h$*=1.5 THz*
- *Line center wavelength:* λ_0*= 5* μm
- *Molecular density (concentration):* $N_{total}= 2.5\times10^{19}\ cm^{-3}$
- *Non-degeneracy factors:* $g_1=5$, $g_2=1$

(a) What is the absorption coefficient $\alpha(cm^{-1})$ at the line center (5 μm) when all the molecules are in their ground state (level 1)? *(12.5 pts.)*

(b) What fraction of the molecules need to be excited into level 2 to make this gas transparent (i.e. the onset of gain) at 5 μm? *(12.5pts.)*

Midterm Formula Sheet (Fall 2012)

PHYC/ECE 464 (Laser Physics I)- University of New Mexico

Instructor: Mansoor Sheik-Bahae

OPTICAL SCIENCE & ENGINEERING
University of New Mexico

Hermite-Gaussian Beams:

$$\frac{E(x,y,z)}{E_0} = H_m\left(\frac{\sqrt{2}x}{w(z)}\right) H_p\left(\frac{\sqrt{2}y}{w(z)}\right) \frac{w_0}{w(z)} \exp\left(-i\frac{kr^2}{2q(z)}\right) \times \exp\left(-i\left[kz-(1+m+p)\tan^{-1}(z/z0)\right]\right)$$

$$\frac{1}{q(z)} = \frac{1}{R(z)} - i\frac{\lambda}{\pi w^2(z)}, \qquad w^2(z) = w_0^2\left(1+\frac{z^2}{z_0^2}\right), \qquad R(z) = z\left(1+\frac{z_0^2}{z^2}\right), \qquad z_0 = \frac{\pi n w_0^2}{\lambda_0}$$

$$k = n\frac{\omega}{c} = \frac{2\pi n}{\lambda_0} \qquad \text{Irradiance: } I = \langle S \rangle = \frac{nc\varepsilon_0}{2}E_0^2 \qquad \text{Snell's Law: } n_i \sin\theta_i = n_t \sin\theta_t$$

Fresnel:

$$r_\parallel = \frac{n_t\cos\theta_i - n_i\cos\theta_t}{n_t\cos\theta_i + n_i\cos\theta_t} = \frac{\tan(\theta_i-\theta_t)}{\tan(\theta_i+\theta_t)} \qquad r_\perp = -\frac{n_i\cos\theta_i - n_t\cos\theta_t}{n_i\cos\theta_i + n_t\cos\theta_t} = -\frac{\sin(\theta_i-\theta_t)}{\sin(\theta_i+\theta_t)}$$

$$t_\parallel = \frac{2n_i\cos\theta_i}{n_i\cos\theta_t + n_t\cos\theta_i} = \frac{2\sin\theta_t\cos\theta_i}{\sin(\theta_i+\theta_t)\cos(\theta_i-\theta_t)} \qquad t_\perp = \frac{2n_i\cos\theta_i}{n_i\cos\theta_i + n_t\cos\theta_t} = \frac{2\sin\theta_t\cos\theta_i}{\sin(\theta_i+\theta_t)}$$

when there is total internal reflection at an air or vacuum interface:

$$r_\parallel = \frac{\cos\theta_i - in_i\sqrt{n_i^2\sin^2\theta_i - 1}}{\cos\theta_i + in_i\sqrt{n_i^2\sin^2\theta_i - 1}} \qquad r_\perp = \frac{n_i\cos\theta_i - i\sqrt{n_i^2\sin^2\theta_i - 1}}{n_i\cos\theta_i + i\sqrt{n_i^2\sin^2\theta_i - 1}}$$

Lens-maker's formula:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Lens Transformation of a Gaussian beam:

$$\frac{1}{R_{out}} = \frac{1}{R_{in}} - \frac{1}{f}$$

Fabry-Perot Transmission and Reflection (general; with gain or absorption):

$$T(\theta, G_0) = \frac{G_0(1-R_1)(1-R_2)}{\left(1-G_0\sqrt{R_1R_2}\right)^2 + 4G_0\sqrt{R_1R_2}\sin^2(\theta)}$$

$$R(\theta, G_0) = \frac{\left(\sqrt{R_1} - G_0\sqrt{R_2}\right)^2 + 4G_0\sqrt{R_1R_2}\sin^2(\theta)}{\left(1-G_0\sqrt{R_1R_2}\right)^2 + 4G_0\sqrt{R_1R_2}\sin^2(\theta)}$$

$$2\Delta\theta_{1/2} = \frac{1-G_0\sqrt{R_1R_2}}{G_0^{1/2}\sqrt[4]{R_1R_2}}$$

$$\theta = kd = \frac{\omega n d}{c}$$

for plane waves

Finesse $F = \frac{\pi\sqrt[4]{R_1R_2}}{1-\sqrt{R_1R_2}} = \frac{\Delta\nu_{FSR}}{\Delta\nu_{1/2}}$

Free Spectral Range: $\Delta\nu_{FSR} = \frac{c}{2nd} = \frac{1}{\tau_{RT}}$

Photon Lifetime:

$$\tau_p = \frac{\tau_{RT}}{1 - R_1R_{21}} \approx \frac{1}{2\pi\Delta\nu_{1/2}}$$

General Resonance Condition:
roundtrip phase change= q2π

Blackbody Radiation (energy density): $\rho(\nu)d\nu = \frac{8\pi n^3 h\nu^3 d\nu}{c^3}\frac{1}{e^{h\nu/kT}-1}$

Lorentzian line shape:
e.g. in natural or pressure broadened

$$g(\nu) = \frac{\Delta\nu_h/2\pi}{(\nu-\nu_0)^2 + (\Delta\nu_h/2)^2}$$

Doppler broadened line shape

$$g(\nu) = \left(\frac{4\ln 2}{\pi}\right)^{1/2}\frac{1}{\Delta\nu_D}\exp\left[(-4\ln 2)\left(\frac{\nu-\nu_0}{\Delta\nu_D}\right)^2\right] \quad \text{with}$$

$$\Delta\nu_D = \left(\frac{8kT\ln 2}{Mc^2}\right)^{1/2}\nu_0$$

Formula Sheet (page 2)
PHYC/ECE 464 (Laser Physics I)- University of New Mexico
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Gain in a two-level system: $\gamma(\nu) = \sigma(\nu)\left[N_2 - \frac{g_2}{g_1}N_1\right]$ Gain cross section: $\sigma(\nu) = A_{21}\frac{\lambda^2}{8\pi n^2}g(\nu)$

Lineshape Normalization: $\int g(\nu)d\nu = 1$ Beer's Law: $\frac{1}{I}\frac{dI}{dz} = -\alpha(I) + \gamma(I)$

Gain or absorption saturation in a homogenously-broadened system:

$\gamma(I) = \frac{\gamma_0}{1 + I/I_s}$ or $\alpha(I) = \frac{\alpha_0}{1 + I/I_s}$ $I_s(\nu) = \frac{h\nu}{\sigma(\nu)\tau_2}$

Einstein's relation: $\frac{A_{21}}{B_{21}} = \frac{8\pi n^3 h\nu^3}{c^3}$ $g_2 B_{21} = g_1 B_{12}$ $\frac{N_2}{N_1} = \frac{g_2}{g_1}e^{-(E_2 - E_1)/kT}$

Degeneracy factors of level i: $g_i = 2J_i + 1$ (J_i is total angular momentum quantum number of that level)

Laser amplifier gain: $\ln\frac{G}{G_0} + \frac{G-1}{I_s/I_{in}} = 0$ where $G_0 = \exp(\gamma_0 L_g)$ is the small-signal gain, $G = I_{out}/I_{in}$

ABCD Matrices $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$ AD-BC=1 $\begin{pmatrix} r_2 \\ r_2' \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} r_1 \\ r_1' \end{pmatrix}$

Free space of length d $\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$	Dielectric interface (from n_1 to n_2) $\begin{pmatrix} 1 & 0 \\ 0 & n_1/n_2 \end{pmatrix}$
Propagation in a medium of length d and index n_2=n immersed in vacuum (n_1=1). $\begin{pmatrix} 1 & d/n \\ 0 & 1 \end{pmatrix}$	Thin lens of focal length f $\begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$
Mirror with radius of curvature R $\begin{pmatrix} 1 & 0 \\ -2/R & 1 \end{pmatrix}$	Spherical dielectric interface $\begin{pmatrix} 1 & 0 \\ (1-n_1/n_2)/R & n_1/n_2 \end{pmatrix}$

ABCD rule for Gaussian beams:

$q_2 = \frac{Aq_1 + B}{Cq_1 + D}$ where

$q(z) = z + iz_0$
or
$\frac{1}{q(z)} = \frac{1}{R(z)} - i\frac{\lambda_0}{\pi n w(z)^2}$

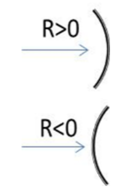
Stability condition: -1<(A+D)/2<1

Laser Threshold: $G_0^2 S = 1$

S= passive cavity survival factor (=$R_1 R_2$ for a simple two mirror cavity)

Photon Density (Photon Number per Volume) $\frac{N_p}{V} = \frac{I}{h\nu c/n_g}$

Convention for refractive and reflective surfaces:



Fundamental Physical Constants

Quantity	Symbol	Value
Speed of light	c	$2.99792458 \ x \ 10^{8} \ m/s$
Planck constant	h	$6.6260755 \ x \ 10^{-34} \ J \cdot s$
Planck constant	h	$4.1356692 \ x \ 10^{-15} \ eV \cdot s$
Planck hbar	$\hbar$	$1.0545727 \ x \ 10^{-34} \ J \cdot s$
Planck hbar	$\hbar$	$6.582121 \ x \ 10^{-16} \ eV \cdot s$
Gravitation constant	G	$6.67259 \ x \ 10^{-11} \ m^{3} \cdot kg^{-1} \cdot s^{-2}$
Boltzmann constant	k	$1.380658 \ x \ 10^{-23} \ J/K$
Molar gas constant	R	$8.314510 \ J/mol \cdot K$
Charge of electron	e	$1.60217733 \ x \ 10^{-19} \ C$
Permeability of vacuum	μ_0	$4\pi \ x \ 10^{-7} \ N/A^{2}$
Permittivity of vacuum	ε_0	$8.854187817 \ x \ 10^{-12} \ F/m$
Mass of electron	m_e	$9.1093897 \ x \ 10^{-31} \ kg$
Mass of proton	m_p	$1.6726231 \ x \ 10^{-27} \ kg$
Mass of neutron	m_n	$1.6749286 \ x \ 10^{-27} \ kg$
Avogadro's number	N_A	$6.0221367 \ x \ 10^{23} \ /mol$
Stefan-Boltzmann constant	σ	$5.67051 \ x \ 10^{-8} \ W/m^{2} \cdot K^{4}$
Rydberg constant	R_∞	$10973731.534 \ m^{-1}$
Bohr magneton	μ_B	$9.2740154 \ x \ 10^{-24} \ J/T$
Bohr radius	a_0	$0.529177249 \ x \ 10^{-10} m$
Standard atmosphere	atm	$101325 \ Pa$

1G = 10^{-4} T , 1 eV = 1.602 × 10^{-19} J, 1 dyne = 10^{-5} N, 1 erg = 10^{-7} J