Laser Physics I (PHY 464)
FALL 2001

NAME ........................................ last
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grade

Please staple and return these pages with your exam.
1. Consider the laser cavity shown below constructed from a plane mirror and a concave mirror of radius $R$. The gain medium (having length $L$ and index of refraction $n$) is placed adjacent to the flat mirror.

(a) Identify a unit cell and obtain the ABCD matrix for a cavity roundtrip. (20 pts.)

(b) Obtain the stability condition in terms of the cavity parameters $R$, $d$, $L$ and $n$. (5 pts.)

(c) Identify the location of the minimum spot size and derive an expression for its magnitude ($w_0$). (10 pts.)

(d) What is the beam curvature at the concave mirror? (3 pts.)

(e) What is the $\Delta v_{\text{FSR}}$? (7 pts.)

(f) What is the cavity finesse and photon lifetime if mirror reflectivities are $R_1$, $R_2$, and the surface reflection (for each surface) of the gain medium is $R_g$. (Note: we are assuming a passive cavity; i.e. no gain) (15 pts.)

(g) Given that $Z_0$ is determined from previous parts, set up the equation for finding the resonant frequency ($v_{\text{res}}$) of a given TEM$_{mn}$ mode. (10 pts.)
2. A two level system is described by a lineshape $g(v)$ shown below and the following parameters:
Spontaneous lifetime ($\tau_p$) = 10 μsec
$\lambda_0$ = 1 μm
$n$ = 2
$N_{\text{total}}$ (= $N_1 + N_2$) = 10$^{18}$ cm$^{-3}$ (at thermal equilibrium all atoms are in level 1)
Degeneracy factors: $g_1 = 1$, $g_2 = 3$

(a) Find the gain (or loss) coefficient at $v = v_0$ if 20% of atoms are excited into level 2.

(b) Find the gain (or loss) coefficient at $v = v_0 - 1$ THz if 80% of atoms are excited into level 2

(15 points)
Some useful formulas:

**Solution for the $q$ (at the starting point) of a cavity:**

$$
\frac{1}{q} = -\frac{A-D}{2B} - i\sqrt{\frac{1 - \left(\frac{A+D}{2}\right)^2}{B}}
$$

**Longitudinal Phase of a Hermite-Gaussian mode:**

$$
\phi(z) = k(z - (1 + m + p)\tan^{-1}(z/z_0))
$$
\[
\begin{align*}
\frac{R}{\mu} + p &= 1 > 0 \\
\Rightarrow \quad \left( \frac{2}{\mu} - 1 \right) &= 1 > \left( \frac{2}{\mu} - 1 \right) > 1 \\
\Rightarrow \quad \left( \frac{2}{\mu} - 1 \right) > 1 > 1
\end{align*}
\]

\[
\begin{align*}
\begin{pmatrix} a & c \\ b & d \end{pmatrix} &= \begin{pmatrix} \frac{x}{d^2 - 1} & 0 \\ 0 & \frac{y}{d^2 - 1} \end{pmatrix} \\
&= \begin{pmatrix} \frac{x}{d^2 - 1} & \frac{y}{d^2 - 1} \\ 0 & \frac{y}{d^2 - 1} \end{pmatrix}
\end{align*}
\]

\[
\begin{align*}
\begin{pmatrix} \frac{x}{d^2 - 1} & \frac{y}{d^2 - 1} \\ 0 & \frac{y}{d^2 - 1} \end{pmatrix} &\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} \frac{x}{d^2 - 1} & \frac{y}{d^2 - 1} \\ 0 & \frac{y}{d^2 - 1} \end{pmatrix}
\end{align*}
\]

\[
\begin{align*}
p + \frac{u}{c} = \frac{d}{c} \\
\Rightarrow \quad \left( \begin{pmatrix} 1 & 0 \\ \frac{\frac{u}{c} - 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{\frac{u}{c} - 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \frac{\frac{u}{c} - 1 & 1 \end{pmatrix}
\end{align*}
\]

\[
\begin{align*}
\begin{pmatrix} 1 & \frac{u}{c} \\ \frac{\frac{u}{c} - 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{\frac{u}{c} - 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{\frac{u}{c} - 1 & 1 \end{pmatrix}
\end{align*}
\]
location of mini spot size at flat mirror such R = ∞ 

Since we choose our starting point of the laser to be at the plane mirror, then the G parameter corresponds to this position.

Note: This is verified by \[ \frac{1}{R} = \frac{A-D}{2B} \]

This \[ \frac{\lambda_0}{\pi W_0^2} = \frac{\sqrt{1 - (1 - \frac{2D}{R})^2}}{2D (1 - \frac{D}{R})} = \frac{\sqrt{4D - 4D^2}}{2D (1 - \frac{D}{R})} = \frac{1}{\sqrt{DR} \sqrt{1 - \frac{D}{R}}} \]

\[ W_0^2 = \frac{\lambda_0}{\pi} \sqrt{DR} \sqrt{1 - \frac{D}{R}} \]

\[ D = d + \frac{L}{\eta} \]

\[ R(\text{at } z = L + d) = R \]

This is the rule of the optical resonator.

\[ \Delta V_{FSR} = \frac{1}{T_{round trip}} = \frac{1}{\frac{L}{c} + \frac{d}{c}} = \frac{c}{d + nL} \]

Comity finesse \[ F = \frac{\pi (R_1 R_2)^{1/4}}{1 - R_1 R_2} \]

But now the Survival factor \[ S = R_1 R_2 \times T_s \]

\[ T_s = (1 - R_s) \] is the transmission of each surface and the beam parameter L emerges in a round trip.

Thus \[ \gamma' = \frac{\pi (R_1 R_2 (1 - R_s)^4)^{1/4}}{1 - \sqrt{R_1 R_2 (1 - R_s)^4}} \]

\[ \gamma' \frac{T_{round trip}}{1 - S} = \frac{(d + nL)c}{1 - R_1 R_2} \]
# 2

\[
A_{21} = \frac{1}{\tau_{50}} = \frac{1}{10 \times 10^{-6}} = 10^5 \text{ sec}^{-1}
\]

\[
\lambda_0 = 1 \text{ mm} = 1 \times 10^{-6} \text{ cm} \quad V_0 = \frac{c}{\lambda_0} = 3 \times 10^{16}
\]

\[
N = 2 \quad g_1 = 1 \quad g_\nu = 3
\]

\[
\sigma(y) = A_{21} \lambda_0^3 g(y) \quad V = \sigma(v) \times (N_2 - \frac{g_1}{g_\nu}, N_1)
\]

\[
\sin \theta \int g(v) dv = 1 \quad \Rightarrow \quad 3 \times \frac{g(v_0)}{2} = 1 \quad \Rightarrow \quad \delta(v_0) = \frac{2}{3} \times 10^{-12} \text{ sec}^{-1}
\]

\[
\sigma(v_0) = 10 \left( \frac{1 \times 10^{-6}}{2} \right) = 6.63 \times 10^{-18} \text{ cm}^2
\]

\[
\gamma(v_0) = 6.63 \times 10^{-18} (0.2 + 3 \times 0.8) 10^{18}
\]

\[
\delta(v_0) = 14.6 \text{ cm}^{-1} \text{ (absorption)} \quad \delta(v, 2 \text{ THz}) = \frac{1}{2} \delta(v_0)
\]

\[
\gamma(v_0 - 1 \text{ THz}) = \frac{6.63 \times 10^{-18} x (0.8 - 3 \times 0.2) 10^{18}}{2} = 0.663 \text{ cm}^{-1}
\]